5 Relations and Functions

BUILDING ON

- writing equations to represent patterns in tables
- graphing and analyzing linear relations

BIG IDEAS

- A relation associates the elements of one set with the elements of another set.
- A function is a special type of relation for which each element of the first set is associated with a unique element of the second set.
- A linear function has a constant rate of change and its graph is a non-vertical straight line.

NEW VOCABULARY

relation

arrow diagram

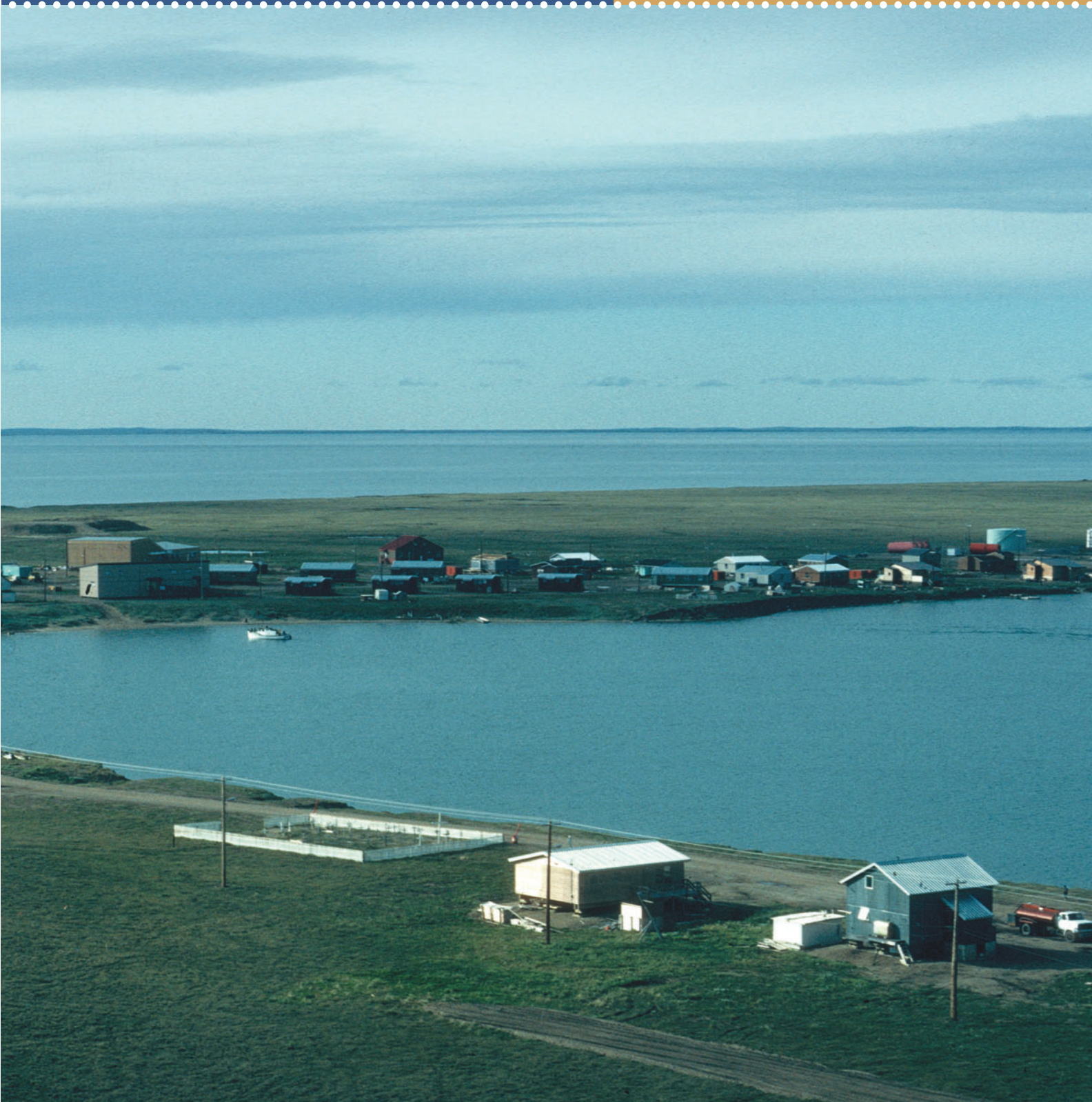function

domain

range

function notation

rate of change

linear function

vertical intercept

horizontal intercept

PAULATUUQ (place of coal) is in the Northwest Territories, north of the Arctic Circle. Here, on June 21st, the longest day of the year, the sun never sets.
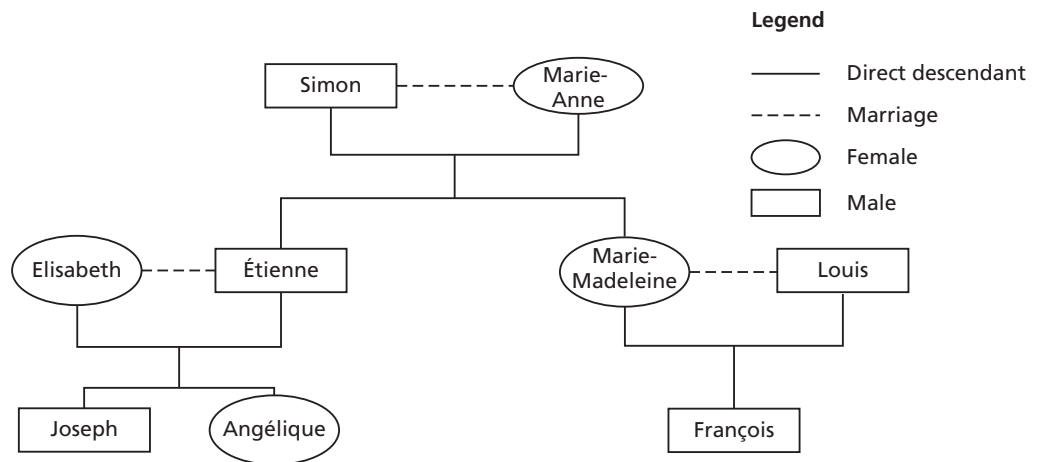
LESSON FOCUS

Represent relations in different ways.

Make Connections

This family tree shows relations within a family.

Legend

——— Direct descendant

- - - - Marriage

◯ Female

▭ Male

How is Joseph related to Simon?

How are Angélique and François related?

How does the family tree show these relations?

Construct Understanding

TRY THIS

A. Use the letters in the square below to write as many words as you can with 3 or more letters.

- Each word must contain the letter in the centre square.
- A letter can only be used once in each word.
- Plurals cannot be used.

```
  S  |  E
   [ T ]
  A  |  R
```
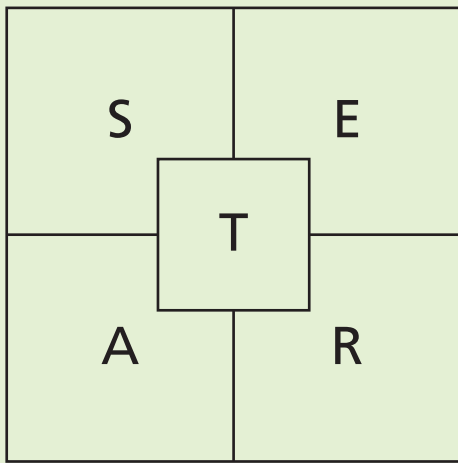
B. Use the letters in the square and the words you wrote.
Pair a letter with a word to show the association "is a letter of the word." Write as many pairs as you can.

C. Use the same association. Suppose you reverse the order of the items in the pairs. Does the association make sense? Explain.

A *set* is a collection of distinct objects.
An *element* of a set is one object in the set.
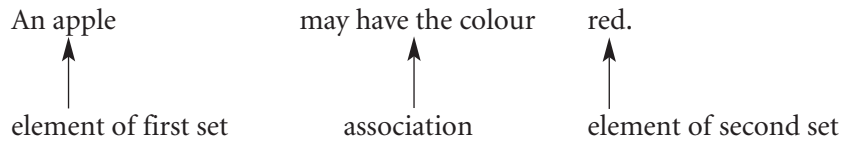A **relation** associates the elements of one set with the elements of another set.

One way to write a set is to list its elements inside braces.
For example, we can write the set of natural numbers from 1 to 5 as:
{1, 2, 3, 4, 5}

The order of the elements in the set does not matter.

Consider the set of fruits and the set of colours.
We can associate fruits with their colours.
For example:

An apple   may have the colour   red.

element of first set   association   element of second set

So, this set of ordered pairs is a relation:

{(apple, red), (apple, green), (blueberry, blue), (cherry, red),
(huckleberry, blue)}

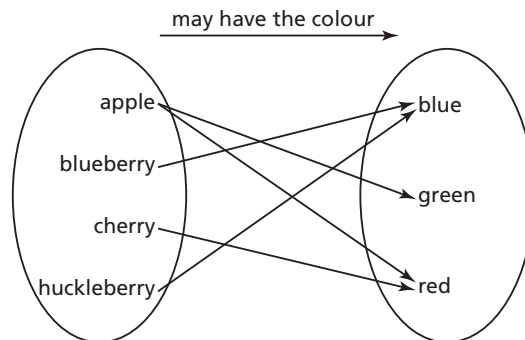Here are some other ways to represent this relation:

■ a table

Fruit	Colour
apple	red
apple	green
blueberry	blue
cherry	red
huckleberry	blue

The heading of each column describes each set.

■ an **arrow diagram**

The two ovals represent the sets.
Each arrow associates an element of the first set with an element
of the second set.

The order of the words in the ordered pairs, the columns in the table, and the
ovals in the arrow diagram is important. It makes sense to say, "an apple may
have the colour red," but it makes no sense to say, "red may have the colour
apple." That is, a relation has direction from one set to the other set.

Example 1   **Representing a Relation Given as a Table**

Northern communities can be associated with the territories they are in. Consider the relation represented by this table.
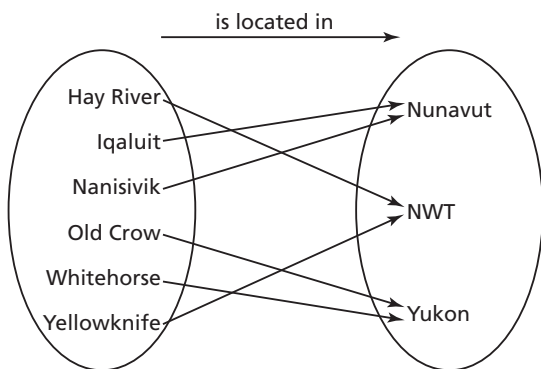
a) Describe this relation in words.

b) Represent this relation:
   i) as a set of ordered pairs
  ii) as an arrow diagram

Community	Territory
Hay River	NWT
Iqaluit	Nunavut
Nanisivik	Nunavut
Old Crow	Yukon
Whitehorse	Yukon
Yellowknife	NWT

SOLUTION

a) The relation shows the association "is located in" from a set of northern communities to a set of territories. For example, Hay River is located in the NWT.

b) i) The communities are the first elements in the ordered pairs. The territories are the second elements in the ordered pairs. The ordered pairs are: {(Hay River, NWT), (Iqaluit, Nunavut), (Nanisivik, Nunavut), (Old Crow, Yukon), (Whitehorse, Yukon), (Yellowknife, NWT)}

  ii) The communities are written in the first set of the arrow diagram.

The territories are written in the second set; each territory is written only once.

CHECK YOUR UNDERSTANDING

1. Animals can be associated with the classes they are in.

Animal	Class
ant	Insecta
eagle	Aves
snake	Reptilia
turtle	Reptilia
whale	Mammalia

a) Describe this relation in words.

b) Represent this relation:
   i) as a set of ordered pairs
  ii) as an arrow diagram

[Answers: a) The relation shows the association "belongs to the class" between a set of animals and a set of classes. b) i) {(ant, Insecta), (eagle, Aves), (snake, Reptilia), (turtle, Reptilia), (whale, Mammalia)}]

Why is the direction of the arrows in the arrow diagram important?

How are the representations the same?

When the elements of one or both sets in a relation are numbers, the relation can be represented as a bar graph.
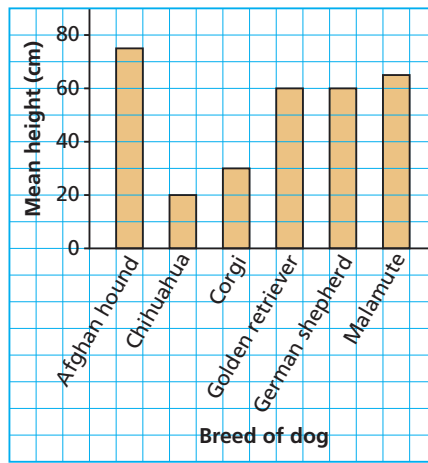
Example 2   Representing a Relation Given as a Bar Graph

Different breeds of dogs can be associated with their mean heights. Consider the relation represented by this graph. Represent the relation:

a) as a table

b) as an arrow diagram

Mean Heights of Different Breeds of Dogs

SOLUTION

a) The association is: "has a mean height of"

In the table, write the breeds of dogs in the first column and the mean heights in centimetres in the second column.

| Breed of Dog	Mean Height (cm)
Afghan hound	75
Chihuahua	20
Corgi	30
Golden retriever	60
German shepherd	60
Malamute	65

b) In the arrow diagram, write the breeds of dogs in the first set and the mean heights in centimetres in the second set.

has a mean height (cm) of
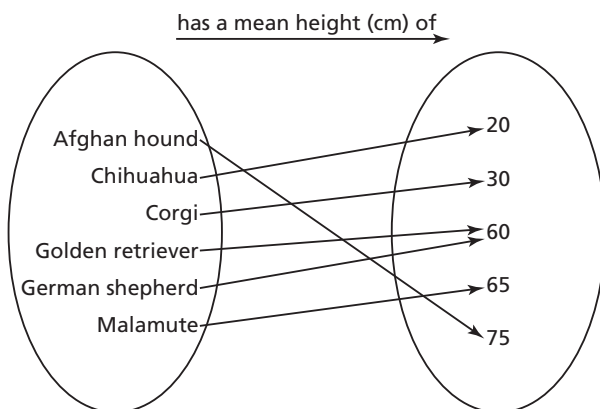
CHECK YOUR UNDERSTANDING

2. Different towns in British Columbia can be associated with the average time, in hours, that it takes to drive to Vancouver. Consider the relation represented by this graph.

Average Travel Time to Vancouver
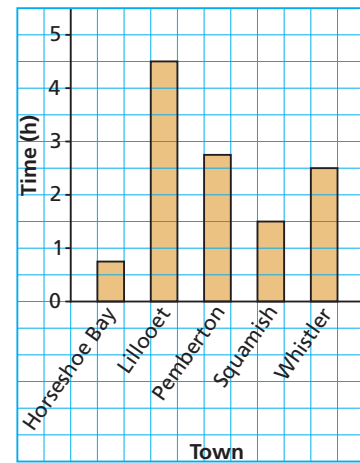
Represent the relation:

a) as a table

b) as an arrow diagram

Answer: a)

Town	Average Time (h)
Horseshoe Bay	0.75
Lillooet	4.5
Pemberton	2.75
Squamish	1.5
Whistler	2.5

Sometimes a relation contains so many ordered pairs that it is impossible to list all of them or to represent them in a table.

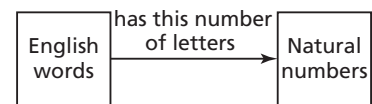## Example 3   Identifying a Relation from a Diagram

In this diagram:

| Western Canadian cities | is within 1-h driving distance of → | Western Canadian cities |

Both sets in a relation can be the same.

a) Describe the relation in words.

b) List 2 ordered pairs that belong to the relation.

SOLUTION

a) The relation shows the association "is within 1-h driving distance" from a set of cities in Western Canada to a set of cities in Western Canada.

b) Two ordered pairs that belong to the relation are:
(Innisfail AB, Olds AB), (Kelowna B.C., Vernon B.C.)

CHECK YOUR UNDERSTANDING

3. In the diagram below:

a) Describe the relation in words.

b) List 2 ordered pairs that belong to the relation.

| English words | has this number of letters → | Natural numbers |

[Sample Answers: a) The relation shows the association "has this number of letters" from a set of English words to a set of natural numbers. b) (mathematics, 11) and (language, 8)]

Discuss the Ideas

1. What are the advantages and disadvantages of the different ways you can represent a relation?

2. Why is the order of the elements in an ordered pair important? Give an example.

Exercises

3. For each table below:
  i) Describe the relation in words.
  ii) Represent the relation:
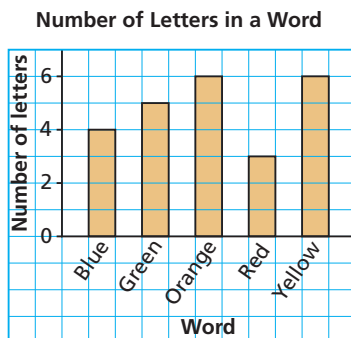   ■ as a set of ordered pairs
    ■ as an arrow diagram

a)

Coin	Value ($)
penny	0.01
nickel	0.05
dime	0.10
quarter	0.25
loonie	1.00
toonie	2.00

b)

Sport	Equipment
badminton	shuttlecock
badminton	racquet
hockey	puck
hockey	stick
tennis	ball
tennis	racquet
soccer	ball

4. Consider the relation represented by this graph.

Number of Letters in a Word

Represent the relation:
a) as a table
b) as an arrow diagram

5. This table shows some of Manitoba's francophone artists and the medium they use.

Artist	Medium
Gaëtanne Sylvester	sculpture
Hubert Théroux	painting
Huguette Gauthier	stained glass
James Culleton	painting
Nathalie Dupont	photography
Simone Hébert Allard	photography

a) Describe the relation in words.
b) Represent this relation:
 i) as a set of ordered pairs
  ii) as an arrow diagram

Burning Sunset Detail by James Culleton

6. a) Describe the relation represented by this bar graph.

**Typical Masses of Species of Salmon**
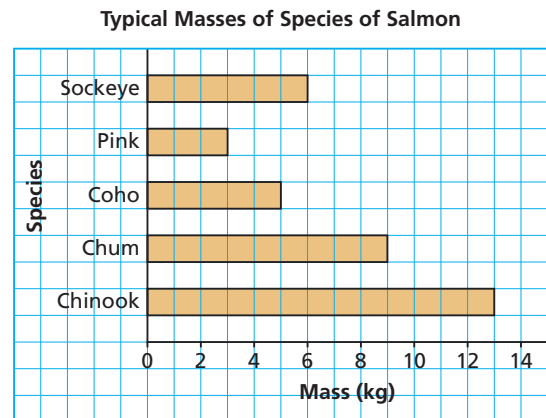
b) Represent the relation as a set of ordered pairs.
c) Represent the relation in a different way.

7. For a word game, words that begin with the letter Z can be difficult to find.
   a) What does this arrow diagram represent?

is the number of letters in
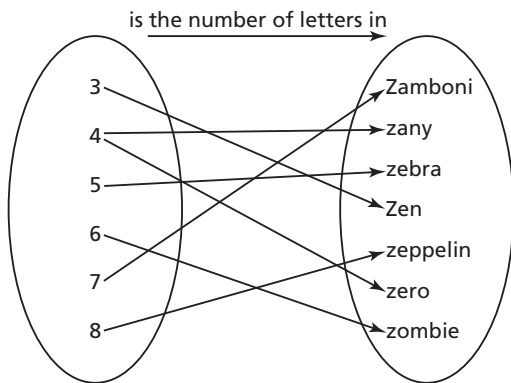
   b) Represent this relation in two different ways.
   c) Create an arrow diagram for words beginning with the letter X, then represent the relation in two different ways.

8. In the diagram below:
   a) Describe the relation in words.
   **b)** List two ordered pairs that belong to the relation.

French words — translates to → English words
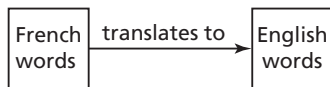
9. A digital clock displays digits from 0 to 9 by lighting up different segments in two squares. For example, the digit 2 needs 5 segments to light up, as shown.
   a) List the set of ordered pairs of the form: (digit, number of segments lit up)
   b) Represent this relation in two different ways.

10. Here are some Canadian hockey players and the year they were born.
   Jennifer Botterill (1979); Jonathan Cheechoo (1980); Roberto Luongo (1979); Jordin Tootoo (1983); Hayley Wickenheiser (1978)
   For each association below, use these data to represent a relation in different ways.
   a) was born in
   b) is the birth year of

11. Choose five people in your class.
   a) Use the association "is older than" to write a relation. Represent the relation using a set of ordered pairs.
   b) Create your own association for these five people, then describe the relation in words. Represent this relation in different ways.

C

12. Two dice are rolled and the numbers that show are recorded.
   a) Use each association below to create a relation as a set of ordered pairs.
       i) The sum of the numbers is even.
       ii) The difference between the numbers is a prime number.
   b) In part a, does the order of the numbers in each ordered pair matter? Explain.

13. The association "is the parent of" is shown in the diagram. Each dot represents a person and each arrow maps a parent to her or his child.
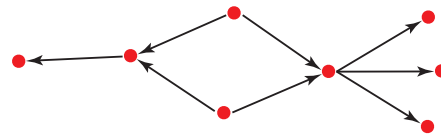
   In this relation:
   a) How many children are shown?
   b) How many parents are shown?
   c) How many grandparents are shown?
   Justify your answers.

14. The association "is the sister of" is shown in the diagram. Each dot represents a person and each arrow maps a sister to a sibling.

   In this relation:
   a) How many females are shown?
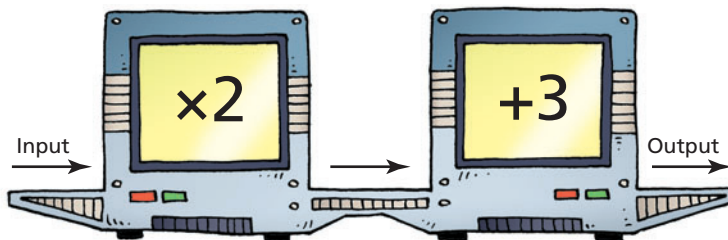   **b)** How many males are shown?
   Justify your answers.

Reflect
Create a relation that you can describe in words. Show two different ways to represent your relation.

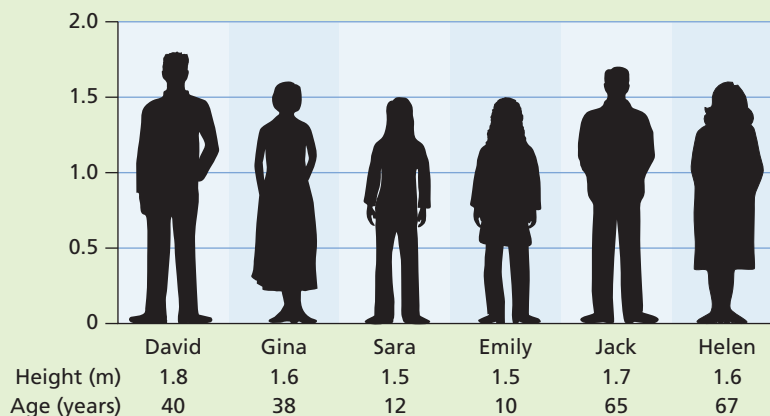# 5.2 Properties of Functions

×2   +3

Input   Output

Make Connections

What is the rule for the Input/Output machine above?
Which numbers would complete this table for the machine?

Input	Output
1	5
2	7
	9
4	
	13

Construct Understanding

THINK ABOUT IT

	David	Gina	Sara	Emily	Jack	Helen
Height (m)	1.8	1.6	1.5	1.5	1.7	1.6
Age (years)	40	38	12	10	65	67

Work in a group of 3.
Use the picture above.

Each of you chooses one of the relations below.

- *name* related to *age*
- *name* related to *height*
- *height* related to *name*

Represent the relation you chose. Compare the relations.
How are they alike? How are they different?

The set of first elements of a relation is called the **domain.**
The set of related second elements of a relation is called the **range.**
A **function** is a special type of relation where each element in the domain is associated with exactly one element in the range.

Here are some different ways to relate vehicles and the number of wheels each has.

This relation associates a number with a vehicle with that number of wheels.

is the number of wheels on a

This relation associates a vehicle with the number of wheels it has.

has this number of wheels
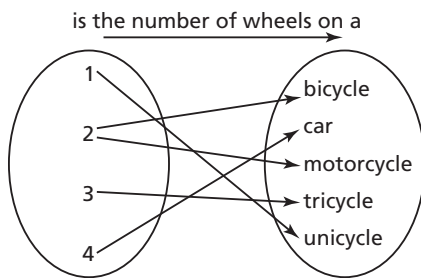
This diagram does not represent a function because there is one element in the first set that associates with two elements in the second set; that is, there are two arrows from 2 in the first set.

{(1, unicycle), (2, bicycle), (2, motorcycle), (3, tricycle), (4, car)}

The set of ordered pairs above does not represent a function because two ordered pairs have the same first element.
The domain is the set of first elements: {1, 2, 3, 4}
The range is the set of associated second elements: {unicycle, bicycle, motorcycle, tricycle, car}

This diagram represents a function because each element in the first set associates with exactly one element in the second set; that is, there is only one arrow from each element in the first set.

{(unicycle, 1), (bicycle, 2), (motorcycle, 2), (tricycle, 3), (car, 4)}

The set of ordered pairs above represents a function because the ordered pairs have different first elements.
The domain is the set of first elements: {unicycle, bicycle, motorcycle, tricycle, car}
The range is the set of associated second elements: {1, 2, 3, 4}

When we list the elements of the range, we do not repeat an element that occurs more than once.

Example 1   Identifying Functions

For each relation below:

■ Determine whether the relation is a function. Justify the answer.

■ Identify the domain and range of each relation that is a function.

a) A relation that associates given shapes with the number of right angles in the shape: {(right triangle, 1), (acute triangle, 0), (square, 4), (rectangle, 4), (regular hexagon, 0)}

b)

is the square of
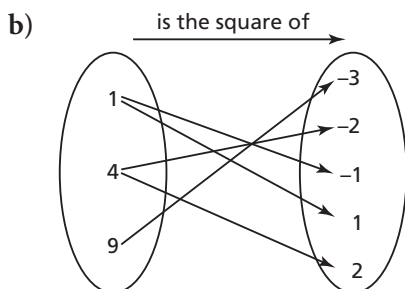
SOLUTION

a) Check to see if any ordered pairs have the same first element: {(right triangle, 1), (acute triangle, 0), (square, 4), (rectangle, 4), (regular hexagon, 0)}
Each ordered pair has a different first element, so for every first element there is exactly one second element. So, the relation is a function.

The domain is the set of the first elements of the ordered pairs: {right triangle, acute triangle, square, rectangle, regular hexagon}
The range is the set of second elements of the ordered pairs: {0, 1, 4}

b) Check to see if any element in the first set associates with more than one element in the second set.

is the square of
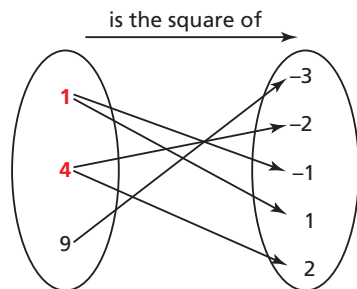
This relation is not a function because each of the numbers 1 and 4 in the first set associates with more than one number in the second set.

CHECK YOUR UNDERSTANDING

1. For each relation below:

■ Determine whether the relation is a function. Justify your answer.

■ Identify the domain and range of each relation that is a function.

a) A relation that associates a number with a prime factor of the number:
{(4, 2), (6, 2), (6, 3), (8, 2), (9, 3)}

b)

has this number of days
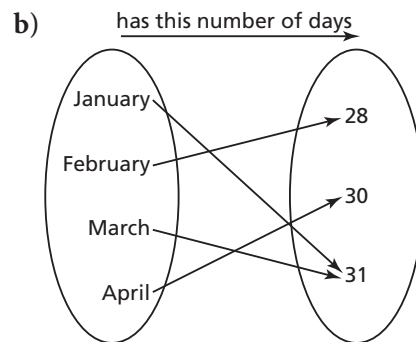
[Answers: a) no b) yes; domain: {January, February, March, April}; range: {28, 30, 31}]

What other strategies could you use to determine whether each relation is a function?

If (*a*, *b*) and (*a*, *c*) are ordered pairs in a function, what can you say about *b* and *c*?

In the workplace, a person's gross pay, P dollars, often depends on the number of hours worked, h.

So, we say P is the *dependent variable*. Since the number of hours worked, h, does not depend on the gross pay, P, we say that h is the *independent variable*.

independent variable ⟶ | **Hours Worked, h** | **Gross Pay, P ($)** | ⟵ dependent variable

domain {

Hours Worked, h	Gross Pay, P ($)
1	12
2	24
3	36
4	48
5	60

} range

A table of values usually represents a sample of the ordered pairs in a relation.

The values of the independent variable are listed in the first column of a table of values. These elements belong to the domain.

The values of the dependent variable are listed in the second column of a table of values. These elements belong to the range.

Example 2   Describing Functions

The table shows the masses, m grams, of different numbers of identical marbles, n.

Number of Marbles, n	Mass of Marbles, m (g)
1	1.27
2	2.54
3	3.81
4	5.08
5	6.35
6	7.62

a) Why is this relation also a function?

b) Identify the independent variable and the dependent variable. Justify the choices.

c) Write the domain and range.

(Solution continues.)

## SOLUTION

a) For each number in the first column, there is only one number in the second column. So, the relation is a function.

b) From an understanding of the situation, the mass of the marbles, m, depends on the number of marbles, n. So, m is the dependent variable and n is the independent variable.

c) The first column of the table is representative of the domain.
The domain is: $\{1, 2, 3, 4, 5, 6,...\}$; that is, all natural numbers

The second column of the table is representative of the range.
The range is: $\{1.27, 2.54, 3.81, 5.08, 6.35, 7.62,...\}$; that is, the product of each natural number and 1.27

We can think of a function as an input/output machine. The input can be any number in the domain, and the output depends on the input number. So, the input is the independent variable and the output is the dependent variable.

Consider two machines that both accept quarters. Machine A calculates the value of the quarters. Machine B weighs the quarters. Each machine performs a different operation, so the machines represent two different functions.

■ Machine A
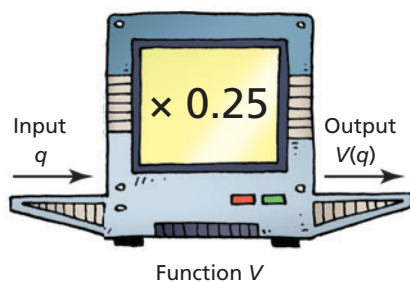
Function V

When the input is q quarters, the output or value, V, in dollars is: $0.25q$
The equation $V = 0.25q$ describes this function.

Since V is a function of q, we can write this equation using **function notation**:
$V(q) = 0.25q$
We say: "V of q is equal to $0.25q$."

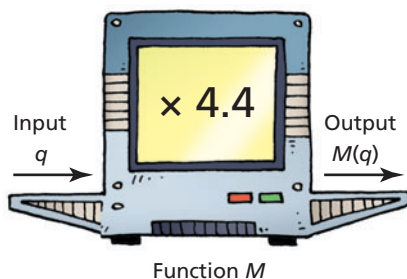This notation shows that V is the dependent variable and that V depends on q.

$V(3)$ represents the value of the function when $q = 3$.

$V(3) = 0.25(3)$

$V(3) = 0.75$

So, the value of 3 quarters is $0.75.

■ **Machine B**

Function *M*

The mass of 1 quarter is 4.4 g.

When the input is q quarters, the output or mass, M, in grams is: $4.4q$
The equation $M = 4.4q$ describes this function.
Since M is a function of q, we can write this equation using function notation:
$M(q) = 4.4q$
This notation shows that M is the dependent variable and that M depends on q.

Any function that can be written as an equation in two variables can be written in function notation. For example, to write the equation $d = 4t + 5$ in function notation, we may write $d(t) = 4t + 5$. t represents an element of the domain and $d(t)$ represents an element of the range.

When we write an equation that is not related to a context, we use x as the independent variable and y as the dependent variable. Then an equation in two variables such as $y = 3x - 2$ may be written as $f(x) = 3x - 2$.

> We can use any other letter, such as g or h or k, to name a function.

Conversely, we may write an equation in function notation as an equation in two variables.
For example, for the equation $C(n) = 300 + 25n$, we write $C = 300 + 25n$.
And, for the equation $g(x) = -2x + 5$, we write $y = -2x + 5$.

Example 3   Using Function Notation to Determine Values

The equation $V = -0.08d + 50$ represents the volume, V litres, of gas remaining in a vehicle's tank after travelling d kilometres. The gas tank is not refilled until it is empty.

a) Describe the function.
   Write the equation in function notation.

b) Determine the value of $V(600)$.
   What does this number represent?

c) Determine the value of d when $V(d) = 26$.
   What does this number represent?

(Solution continues.)

CHECK YOUR UNDERSTANDING

3. The equation $C = 25n + 1000$ represents the cost, C dollars, for a feast following an Arctic sports competition, where n is the number of people attending.

   a) Describe the function.
      Write the equation in function notation.

(Question continues.)

SOLUTION

a) The volume of gas remaining in a vehicle's tank is a function of the distance travelled. In function notation:
$$V(d) = -0.08d + 50$$

b) To determine $V(600)$, use:

$V(d) = -0.08d + 50$ *Substitute: $d = 600$*

$V(600) = -0.08(600) + 50$

$V(600) = -48 + 50$

$V(600) = 2$

$V(600)$ is the value of V when $d = 600$.
This means that when the car has travelled 600 km, the volume of gas remaining in the vehicle's tank is 2 L.

c) To determine the value of d when $V(d) = 26$, use:

$V(d) = -0.08d + 50$ *Substitute: $V(d) = 26$*

$26 = -0.08d + 50$ *Solve for d.*

$-24 = -0.08d$ *Divide each side by -0.08.*

$d = 300$

$V(300) = 26$ means that when $d = 300$, $V = 26$; that is, after the car has travelled 300 km, 26 L of gas remains in the vehicle's tank.

b) Determine the value of $C(100)$. What does this number represent?

c) Determine the value of n when $C(n) = 5000$. What does this number represent?

[Answers: a) $C(n) = 25n + 1000$
b) $3500 c) 160]

> What values of d do not make sense as possible domain values?

Discuss the Ideas

1. How can you tell whether a set of ordered pairs represents a function?

2. When a function is completely represented using a set of ordered pairs or a table of values, how can you determine the domain and range of the function?

3. Why are some relations not functions? Why are all functions also relations?

Exercises

A

4. Which arrow diagrams represent functions?

a)
doubled is

b)
is greater than

c)
squared is
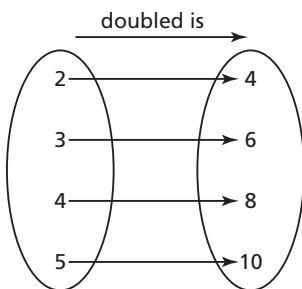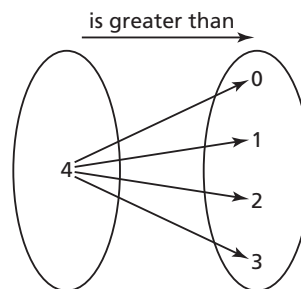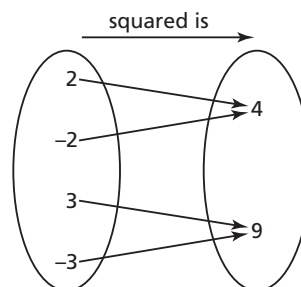
5. Which sets of ordered pairs represent functions? Identify the domain and range of each set of ordered pairs.
a) {(1, 3), (2, 6), (3, 9), (4, 12)}
b) {(1, 0), (0, 1), (−1, 0), (0, −1)}
c) {(2, 3), (4, 5), (6, 7), (8, 9)}
d) {(0, 1), (0, 2), (1, 2), (0, 3), (1, 3), (2, 3)}

6. Write in function notation.
a) $C = 20n + 8$
b) $P = n - 3$
c) $t = 5d$
d) $y = -x$

7. Write as an equation in two variables.
a) $d(t) = 3t - 5$
b) $f(x) = -6x + 4$
c) $C(n) = 5n$
d) $P(n) = 2n - 7$

B

8. For each relation below:
- Determine whether the relation is a function. Justify your answer.
- Identify the domain and range of each relation.

a) {(1, 1), (2, 8), (3, 27), (4, 64)}
b) {(3, 4), (3, 5), (3, 6), (3, 7)}

9. For each table of values below:
i) Explain why the relation is a function.
ii) Identify the independent variable and the dependent variable. Justify your choices.
iii) Write the domain and range.

a)

Number of Cans of Juice Purchased, n	Cost, C ($)
1	2.39
2	4.00
3	6.39
4	8.00
5	10.39
6	12.00

b)

Altitude, A (m)	Temperature, T (°C)
610	15.0
1220	11.1
1830	7.1
2440	3.1
3050	−0.8
3660	−4.8

10. This set of ordered pairs associates a number with a polygon that has that number of sides:
{(3, isosceles triangle), (3, equilateral triangle), (3, right triangle), (3, scalene triangle), (4, square), (4, rectangle), (4, rhombus), (4, trapezoid), (4, parallelogram), (5, pentagon), (6, hexagon)}
a) Does the set of ordered pairs represent a function? Explain.
b) Suppose the elements in the ordered pairs were reversed. Use the association "has this number of sides." Would the new relation be a function? Explain.
c) Identify the domain and range of each relation in parts a and b.

11. The Rassemblement jeunesse francophone in Alberta brings together French language high school students from all over the province for a day of activities. Use two columns in this table to represent a relation.
a) Name two relations that are functions.
b) Name two relations that are not functions. Justify your answers.

Name	From	Age	Gender
Marie	Edmonton	13	F
Gabriel	Falher	16	M
Élise	Bonnyville	14	F
Christophe	Calgary	13	M
Jean	Edmonton	15	M
Mélanie	Edmonton	15	F
Nicole	Red Deer	17	F
Marc	Légal	13	M

12. Which statement is true? Give an example to justify your choice.

    a) All functions are relations, but not all relations are functions.

    b) All relations are functions, but not all functions are relations.

13. In a crossword game, each letter is worth a certain number of points. Here are some letters and their points.

    a) Create two different tables to represent relations that associate these letters and their points.

    b) Which table in part a represents a function? Justify your choice.

14. For the function $f(x) = -5x + 11$, determine:

    a) $f(1)$ **b)** $f(-3)$

    c) $f(0)$ **d)** $f(1.2)$

15. **a)** For the function $f(n) = 2n - 7$, determine n when:

        i) $f(n) = 11$ **ii)** $f(n) = -6$

    b) For the function $g(x) = -5x + 1$, determine x when:

        i) $g(x) = 41$ **ii)** $g(x) = -16$

16. The function $C(i) = 2.54i$ converts a measurement of i inches to a measurement of C centimetres.

    a) Write the function as an equation in 2 variables.

    b) Determine the value of $C(12)$. What does this number represent?

    c) Determine the value of i when $C(i) = 100$. What does this number represent?

17. A car is travelling toward Meadow Lake Park, Saskatchewan. The equation $D = -80t + 300$ represents the distance, D kilometres, to Meadow Lake after t hours of driving.

    a) Describe the function. Write this equation in function notation.

    b) How far away from Meadow Lake Park was the car at the start of its journey? How do you know?

18. Anthropologists who study human remains have developed equations for estimating the height of a person from a measure of her or his bones. The height in centimetres is a function of the length, l centimetres, of the humerus (the upper arm bone).

    For a female: $f(l) = 2.754l + 71.475$
    For a male: $m(l) = 2.894l + 70.641$

    a) Determine each value. What does each number represent?

        i) $f(15)$ **ii)** $m(20)$

    b) Determine each value of l. What does each number represent?

        i) $f(l) = 142$ **ii)** $m(l) = 194$

    c) Measure the length of your humerus. Use an equation to estimate your height. How close was your answer to your actual height?

19. The function $C(f) = \dfrac{5}{9}(f - 32)$ converts a temperature, f degrees Fahrenheit, to C degrees Celsius.

    a) Determine:

        i) $C(50)$ **ii)** $C(-13)$

    b) Determine each value of f when:

        i) $C(f) = 20$ **ii)** $C(f) = -35$

    c) Write an equation in function notation to relate the temperatures in each fact.

        i) Pure water freezes at 0°C or 32°F.

        ii) Pure water boils at 100°C or 212°F.

        iii) Cookies are baked at 180°C or 356°F.

20. To convert a temperature in degrees Celsius to degrees Fahrenheit, multiply the Celsius temperature by $\frac{9}{5}$ then add 32. Use these instructions to write an equation in function notation for this conversion.

21. The area of a rectangle with length l centimetres and width w centimetres is 9 cm². Express the perimeter of the rectangle as a function of its length.

22. A rectangle with length l centimetres and width w centimetres has a perimeter of 12 cm. Use function notation to express the length of the rectangle as a function of its width. What are the domain and range of the function?

23. The lengths of the sides of a triangle, in units, are s, $s + 5$, and t. Its perimeter is 16 units. Use function notation to express t as a function of s. What are the domain and range of the function?

Reflect   Describe how you can determine if a relation is a function. Use an example to illustrate each strategy you might use.

THE WORLD OF MATH

Careers: Forensic Anthropologist

Forensic anthropologists study human remains to understand more about how people develop, both as individuals and as societies. They collect data on bones and teeth to identify the sex, height, mass, race, and age at death. Forensic anthropologists work in crime labs, law enforcement agencies, museums, or at archaeological sites; and may give expert testimony in court. They may identify bones and bone fragments that have been in storage for many years.

humerus
ulna
radius

Specimens must be cleaned, accurately measured, and catalogued. Measurements of the skull and teeth can help to estimate the age of a person. When the length of the humerus, radius, or ulna is known, then a person's height can be approximated.

Here are some typical equations used to estimate the height of a person from the length of her or his radius.

For a female: $h = 3.343r + 81.224$
For a male: $h = 3.271r + 89.925$,

where h is the height of the person in centimetres and r is the length of the radius in centimetres.
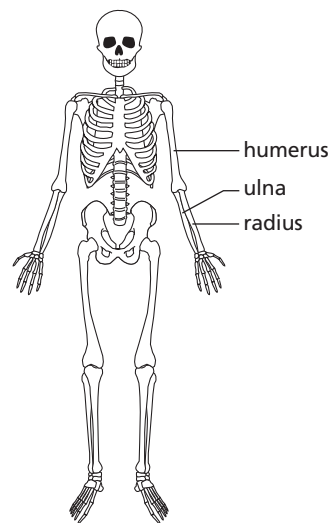
Connections

Here is a Frayer model for a function.

Definition

A function is a relation where each element in the first set is associated with exactly one element in the second set.

Essential Characteristics

The domain is the set of first elements in the ordered pairs. These are the values of the independent variable. The range is the set of second elements in the ordered pairs. These are the values of the dependent variable.

Function

Example

x	y
0	4
1	5
2	6
3	7

{(0, 4), (1, 5), (2, 6), (3, 7)}

plus 4 equals

Non-examples

Number of Faces	Object
4	triangular pyramid
5	square pyramid
6	cube
6	rectangular prism

{(4, triangular pyramid),
(5, square pyramid),
(6, cube),
(6, rectangular prism)}

is the number of faces on a

Concept Development

■ **In Lesson 5.1**

– You described a relation in words and represented it using: a set of ordered pairs, an arrow diagram, a table, and a bar graph.

■ **In Lesson 5.2**

– You identified a function by checking to see whether its ordered pairs had different first elements.

– You listed the elements of the domain and of the range.

– You related the elements of the domain to the independent variable and the elements of the range to the dependent variable.

– You described functions in words, and algebraically using function notation.

Assess Your Understanding

1. Copy and complete this table for different representations of relations.

	Description in Words	Set of Ordered Pairs	Arrow Diagram	Table or Graph
a)		{(skin, drum), (skin, kayak), (bark, basket), (stone, inukshuk), (stone, carving)}		
b)				(table of Number / Number of Factors)
c)			is usually coloured (arrow diagram: grass, sea, sky, snow → blue, green, white)	
d)	For the numbers 1 to 4, the first number in an ordered pair is greater than the second number.			

Table for part b):

Number	Number of Factors
1	1
2	2
3	2
4	3

2. a) Which relations in question 1 are functions? Justify your answers.
   b) State the domain and range of each function.

3. a) Think about two sets of numbers and an association.
   i) Create a relation that is not a function.
   ii) Create a function.
   b) Represent each relation in part a in different ways.

4. The temperature, T degrees Celsius, of Earth's interior is a function of the distance, d kilometres, below the surface: $T(d) = 10d + 20$
   a) Identify the dependent and independent variables.
   b) Write this function as an equation in two variables.
   c) Determine the value of $T(5)$. Describe what this number represents.
   d) Determine the value of d when $T(d) = 50$. Describe what this number represents.

5.3 Interpreting and Sketching Graphs

LESSON FOCUS

Describe a possible situation for a given graph and sketch a possible graph for a given situation.

Make Connections

In math, a graph provides much information.
This graph shows the depth of a scuba diver as a function of time.

A Scuba Diver's Dive
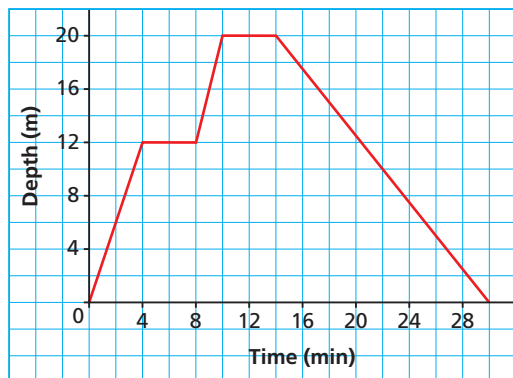
How many minutes did the dive last?

At what times did the diver stop her descent?

What was the greatest depth the diver reached? For how many minutes was the diver at that depth?

Construct Understanding

TRY THIS

Work with a partner.
You will need grid paper.

This graph shows the depth of water in a bathtub as a function of time.

Depth of Water in a Bathtub
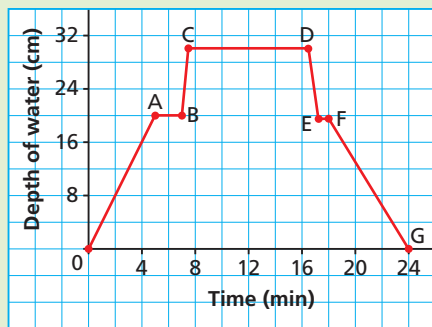
A. What does each segment of the graph represent? Compare your description with that of your partner. Are both your stories the same? Should they be? Explain.

B. Sketch a graph to represent this situation:
You put the plug in the bath and turn on the taps.
You leave the bathroom and return to discover that the bath has overflowed.
You turn off the taps and pull out the plug to let out some water. You put the plug back in.

C. Compare your graph with that of your partner. How are the graphs the same? How are they different?

The properties of a graph can provide information about a given situation.
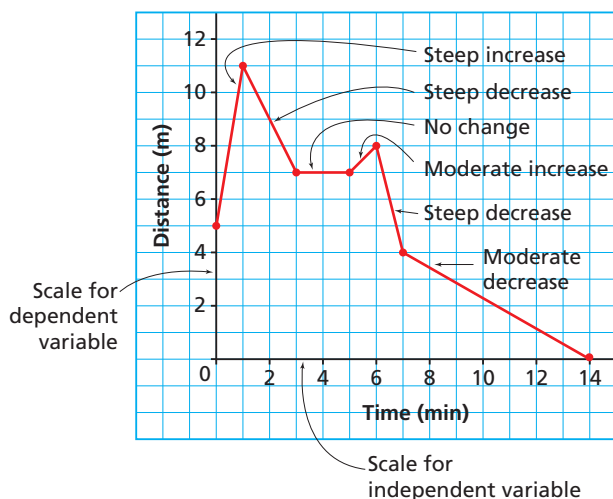
Example 1 Interpreting a Graph

Each point on this graph represents a bag of popping corn. Explain the answer to each question below.

a) Which bag is the most expensive? What does it cost?

b) Which bag has the least mass? What is this mass?

c) Which bags have the same mass? What is this mass?

d) Which bags cost the same? What is this cost?

e) Which of bags C or D has the better value for money?

Costs and Masses of Various Bags of Popcorn
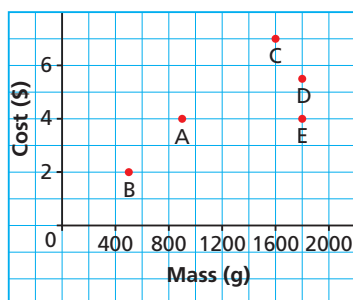
SOLUTION

a) Bag C is most expensive because it is represented by the highest point on the graph and the vertical axis represents cost. It costs $7.00.

b) Bag B has the least mass because it is represented by the point on the graph farthest to the left and the horizontal axis represents mass. The mass appears to be 500 g.

c) Bags D and E have the same mass because the points that represent them lie on the same vertical line and it passes through 1800 on the *Mass* axis. The mass is 1800 g.

d) Bags A and E cost the same because the points that represent them lie on the same horizontal line and it passes through 4 on the *Cost* axis. The cost is $4.00.

e) Bag D has the better value for money because it has a greater mass than bag C and costs less than bag C.

CHECK YOUR UNDERSTANDING

1. Each point on this graph represents a person. Explain your answer to each question below.

Ages and Heights of People
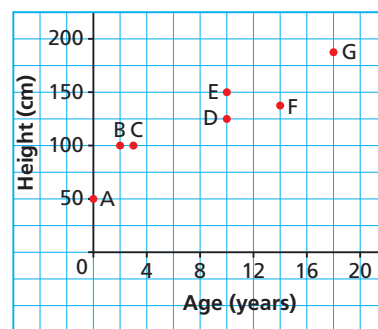
a) Which person is the oldest? What is her or his age?

b) Which person is the youngest? What is her or his age?

c) Which two people have the same height? What is this height?

d) Which two people have the same age? What is this age?

e) Which of person B or C is taller for her or his age?

[Answers: a) G, 18 years
b) A, newborn
c) B and C, 100 cm
d) D and E, 10 years
e) B]

Does this graph represent a function? Explain.

Why do you think bag D is more expensive than bag E?

The graph shows how the volume of water in a watering can changes over time.

The starting volume is 1 L, which is the volume at point A.
Segment AB goes up to the right, so the volume of water is increasing from 0 s to 30 s.
Segment BC is horizontal, so the volume is constant from 30 s to 70 s.
Segment CD goes down to the right, so the volume is decreasing from 70 s to 90 s.
At point D, the volume is 0 L after 90 s.

Volume of Water in a Watering Can
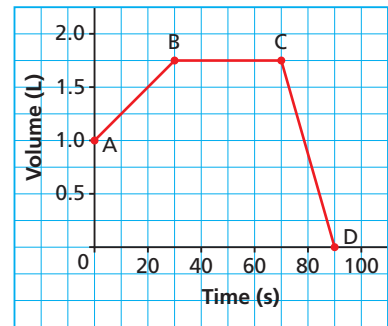
Example 2 — Describing a Possible Situation for a Graph

Describe the journey for each segment of the graph.

Day Trip from Winnipeg to Winkler, Manitoba
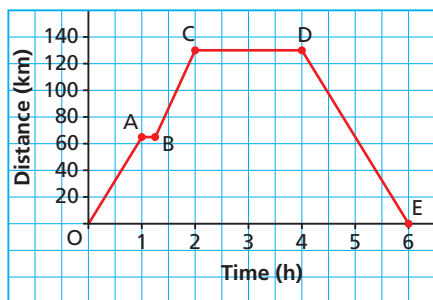
The distance between Winnipeg and Winkler is 130 km.

SOLUTION

Segment	Graph	Journey
OA	The graph goes up to the right, so as time increases, the distance from Winnipeg increases.	In the first hour, the car leaves Winnipeg and travels approximately 65 km toward Winkler.
AB	The graph is horizontal, so as time increases, the distance stays the same.	The car stops for approximately 15 min.
BC	The graph goes up to the right, so as time increases, the distance increases.	The car travels approximately 65 km toward Winkler.
CD	The graph is horizontal, so as time increases, the distance stays the same.	At C, the car has travelled 130 km so it has reached Winkler, where it stops for 2 h.
DE	The graph goes down to the right, so as time increases, the distance decreases.	The car returns to Winnipeg and takes 2 h to travel 130 km.

CHECK YOUR UNDERSTANDING

2. This graph represents a day trip from Athabasca to Kikino in Alberta, a distance of approximately 140 km.

   Describe the journey for each segment of the graph.

   Day Trip from Athabasca to Kikino
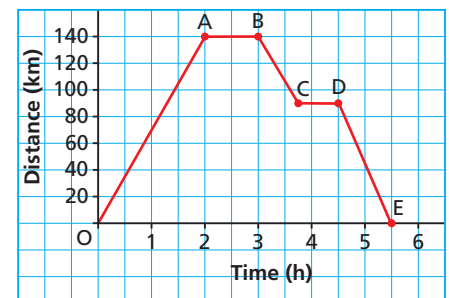
   [Answer: The car takes 2 h to travel 140 km to Kikino; the car stops for 1 h: the car takes approximately 45 min to travel 50 km toward Athabasca; the car stops for approximately 45 min; the car takes 1 h to travel approximately 90 km to Athabasca]

   What was the total driving time? Explain.

   What are the dependent and the independent variables?

Example 3   **Sketching a Graph for a Given Situation**

Samuel went on a bicycle ride. He accelerated until he reached a speed of 20 km/h, then he cycled for 30 min at approximately 20 km/h. Samuel arrived at the bottom of a hill, and his speed decreased to approximately 5 km/h for 10 min as he cycled up the hill. He stopped at the top of the hill for 10 min.

Sketch a graph of speed as a function of time. Label each section of the graph, and explain what it represents.

SOLUTION

Draw and label axes on a grid. The horizontal axis represents time in minutes. The vertical axis represents speed in kilometres per hour.

Samuel's Bicycle Ride
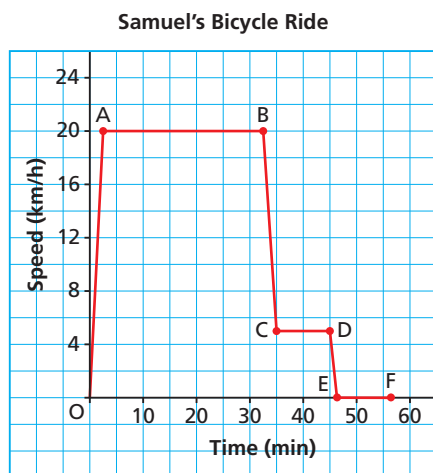
Segment	Journey
OA	Samuel's speed increases from 0 to 20 km/h, so the segment goes up to the right.
AB	Samuel cycles at approximately 20 km/h for 30 min. His speed does not change, so the segment is horizontal.
BC	Samuel's speed decreases to 5 km/h, so the segment goes down to the right.
CD	Samuel cycles uphill at approximately 5 km/h for 10 min. His speed does not change, so the segment is horizontal.
DE	Samuel slows down to 0 km/h, so his speed decreases and the segment goes down to the right.
EF	Samuel remains stopped at 0 km/h for 10 min, so the segment is horizontal.

CHECK YOUR UNDERSTANDING

3. At the beginning of a race, Alicia took 2 s to reach a speed of 8 m/s. She ran at approximately 8 m/s for 12 s, then slowed down to a stop in 2 s. Sketch a graph of speed as a function of time. Label each section of your graph, and explain what it represents.

Answer:

Alicia's Race
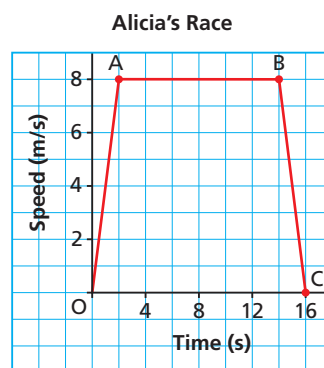
Why does the graph not end at point E?

1. For a graph of distance as a function of time, what does each segment represent?
   - a horizontal line segment
   - a segment that goes up to the right
   - a segment that goes down to the right

2. For a graph of speed as a function of time, what does each segment represent?
   - a horizontal line segment
   - a segment that goes up to the right
   - a segment that goes down to the right

Exercises

A

3. Each point on the graph represents a polar bear. Explain the answer to each question below.

Heights and Masses of 8 Polar Bears
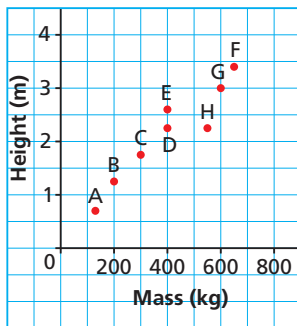
a) Which bear has the greatest mass? What is this mass?
b) Which bear is the shortest? What is its height?
c) Which two bears have the same mass? What is this mass?
d) Which two bears have the same height? What is this height?

B

4. This graph shows the height of the tide in a harbour as a function of time in one day. Explain the answer to each question below.

Height of the Tide in a Harbour
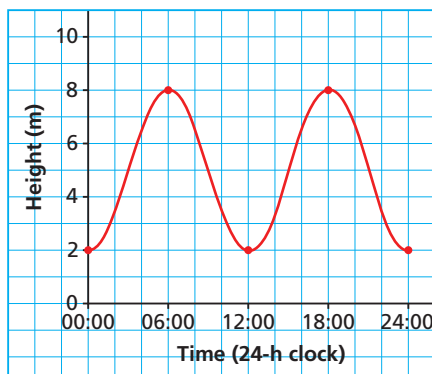
a) What is the greatest height? At what times does it occur?
b) What is the least height? At what times does it occur?
c) How high is the tide at 04:00?
d) When is the tide 4 m high?

5. To raise a flag, Sepideh pulls the rope steadily with both hands for a short time, then moves both hands up the rope and pulls again. She does this until the flag has been raised. Which graph best represents the height of the flag? Give reasons for your choice.

Graph A
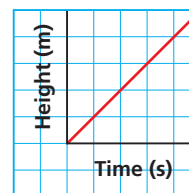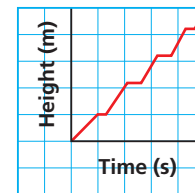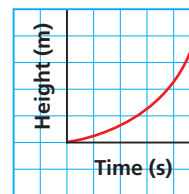
Graph B
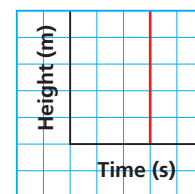
Graph C

Graph D

6. Gill runs for exercise. This graph shows her distance from home during one of her runs. Describe Gill's run for each segment of the graph.

Gill's Run

5.3 Interpreting and Sketching Graphs **281**
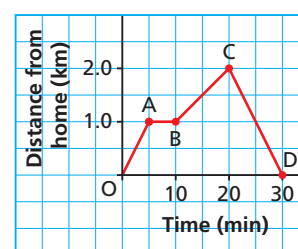
7. Katanya went scuba diving in Egypt. This graph shows her depth below sea level as a function of time on one of her dives.

Depth of Katanya's Scuba Dive
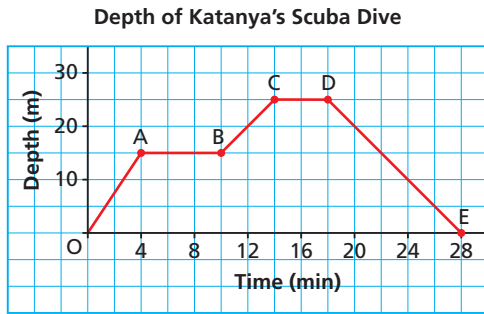
Write all that you know about the dive from the graph.

8. Point A and Point B represent the same helicopters in each of these graphs.

Graph of Cost against Age
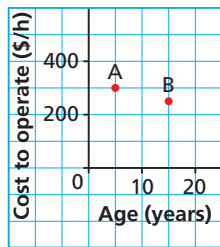
Graph of Maximum Speed against Number of Seats
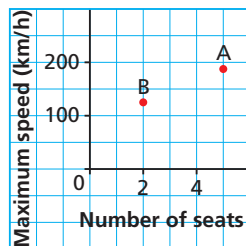
Which statements are true? Justify your answers.

a) The older helicopter is cheaper to operate.

b) The helicopter with more seats has the lower maximum speed.

c) The helicopter with the lower maximum speed is cheaper to operate.

d) The helicopter with the greater maximum speed is older.

e) The helicopter with fewer seats is newer.

9. a) Describe what is happening for each line segment in this graph.

Volume of Gas in a Snowmobile
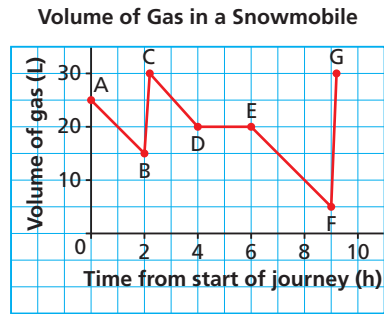
b) How much gas was in the tank at the start of the journey? Was the tank full at this time? Explain.

10. An oven is turned on at a room temperature of 20°C and it takes 10 min to reach a temperature of 190°C. A tray of cookies is placed in the oven to bake for 10 min. The oven is then turned off and returns to room temperature after 15 min. Sketch a graph of temperature as a function of time. Label each section of the graph and explain what it represents.

11. Write all that you know about a person's drive to work from this graph.

A Person's Drive to Work
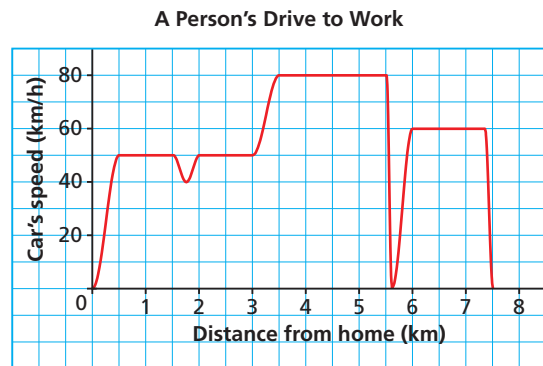
12. A school vending machine sells cartons of milk and juice. On a typical day:
- No cartons are sold between 7 A.M. and 8 A.M., or from 5 P.M. onward.
- The machine has a capacity of 100 cartons. At 7 A.M., it is three-quarters full.
- From 8 A.M. to 10 A.M., 10:15 A.M. to noon, and from 1 P.M. to 3 P.M. the students are in class.
- The machine is filled at 11 A.M. and at 4 P.M.

Sketch a graph of the number of cartons in the vending machine as a function of time. Explain what each section of the graph represents.

13. A student drew a graph to represent this situation.

"Jonah is watching television. After 3 min his mom enters the room to ask him a question. He turns the volume down a bit, answers his mom, then turns the volume back up. Two minutes later, Jonah's dad turns on the dishwasher so Jonah gradually turns up the volume. After a further 3 min, a commercial comes on so Jonah presses the mute button."

Describe any errors in the student's graph.

Volume of a Television
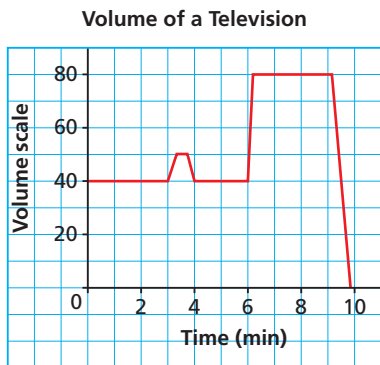
14. The two graphs below have the same shape, but different vertical axes. Copy each graph and include numbers and units on both axes. Write and justify a possible situation that it represents.

a) Graph A
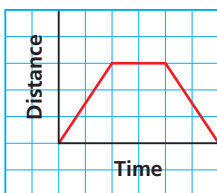
b) Graph B
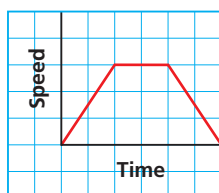
15. Each graph shows a quantity as a function of time. Choose a dependent variable for each graph, and suggest a possible situation that it represents. Copy the graph and include numbers on the axes.

a)

b)
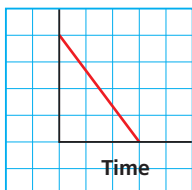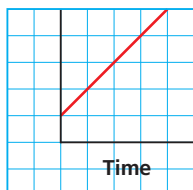
16. Chad goes bungee jumping.
   a) Sketch each graph of his jump.
      i) the distance above the ground as a function of time
      ii) the speed as a function of time
   b) Explain the similarities and differences between the two graphs.

17. For each graph, choose a dependent variable and an independent variable, and suggest a possible situation that it represents. Describe the significance of any key points or changes in the graph.

a)
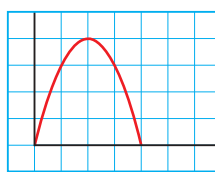
b)
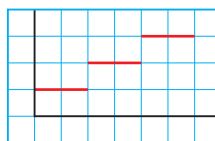
c)
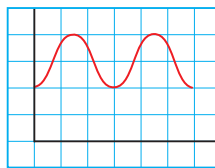
18. The diagrams below show cross-sections of swimming pools that will be filled with water at the same constant rate. Sketch two graphs on the same grid to represent the depth of water in each pool as a function of time. Label the axes. Justify the shape of each graph.

a)

Fill Here

b)

Fill Here
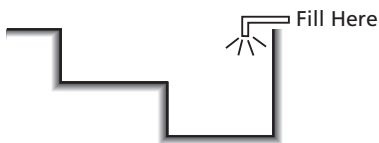
Reflect

When you describe a possible situation for a given graph, what features of the graph do you have to use in your description and how do you use them? Include a sketch of a graph in your explanation.

LESSON FOCUS

Graph data and investigate the domain and range when the data represent a function.

Make Connections

To rent a car for less than one week from Ace Car Rentals, the cost is $65 per day for the first three days, then $60 a day for each additional day.

Number of Days Car Is Rented	Total Cost ($)
1	65
2	130
3	195
4	255
5	315
6	375
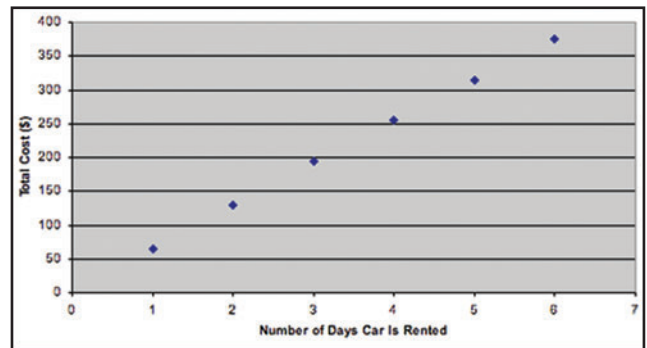
Why are the points on the graph not joined?

Is this relation a function? How can you tell?

What is the domain? What is the range?

Construct Understanding

TRY THIS

Work with a partner.

You will need:
- a length of rope
- a metre stick
- grid paper, a graphing calculator, or a computer with graphing software

When you tie knots in a rope, the length of the rope is related to the number of knots tied.

A. You will investigate the relation between the number of knots and the length of rope.
- Measure the length of the rope without any knots. Tie a knot in the rope. Measure the length of the rope with the knot.
- Repeat the measurements for up to 5 knots. Try to tie the knots so they are all the same size and tightness.
- Record the number of knots and the length of the rope in a table.

Number of Knots	Length of Rope (cm)

B. Graph the data.
- How did you determine on which axis to plot each variable?
- How did you choose the scale for each axis so the data fit on the axes?
- Did you join the points? Justify your answer.

C. If you used a graphing calculator, sketch the graph. If you used a computer, print the graph.
- Is the length of the rope a function of the number of knots? Explain. If your answer is yes, list the set of ordered pairs. What is the domain? What is the range?
- Would it make sense to extend the graph to the right? To the left? If your answer is yes, how far could you extend it? What is the new domain? What is the new range? If your answer is no, what restrictions are there on the domain and range?

Suppose you used another piece of rope with the same length. Would the length of the rope after each number of knots be the same as that which you first recorded? Why or why not?

D. Suppose you combined your data with those of 4 pairs of classmates.

- When you graph all the data, does the graph represent a function? Justify your answer.

- Suppose you calculated the mean rope length for each number of knots, then graphed the data. Would the graph represent a function? Justify your answer.

Assess Your Understanding

1. For each table of values below:

  i) Graph the data. Will you join the points? Justify your answer.
  ii) Does the graph represent a function? Explain.

a) At a constant pressure, the speed of sound in air is related to the air temperature.

Air Temperature (°C)	Speed of Sound (m/s)
0	331
5	334
10	337
15	340
20	343

b) The recommended daily dose of vitamin C is related to a female's age in years.

Age (years)	Dose of Vitamin C Tablet (mg)
3	15
6	25
9	45
12	45
15	65
18	65
21	75

2. Graph the data in these tables of values from Lesson 5.2, question 9. Decide whether to join the points. How can you tell from each graph that the relation is a function?

a)

Number of Cans of Juice Purchased, n	Cost, C ($)
1	2.39
2	4.00
3	6.39
4	8.00
5	10.39
6	12.00

b)

Altitude, A (m)	Temperature, T (°C)
610	15.0
1220	11.1
1830	7.1
2440	3.1
3050	−0.8
3660	−4.8

5.5 Graphs of Relations and Functions

The great horned owl is Alberta's provincial bird.

Make Connections

In an environmental study in Northern Alberta, Joe collected data on the numbers of different species of birds he heard or saw in a 1-h period every 2 h for 24 h. Alice collected data on the temperature in the area at the end of each 1-h period. They plotted their data:
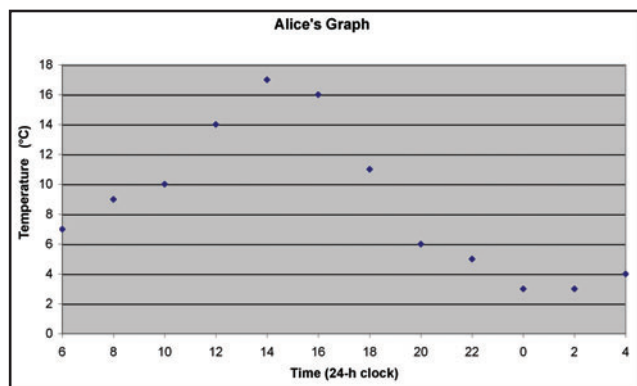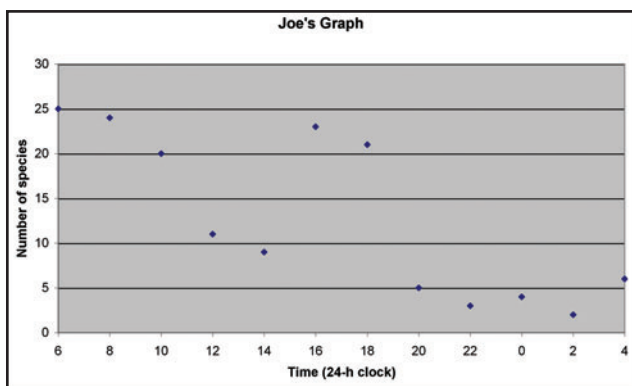
Does each graph represent a relation? A function? How can you tell?

Which of these graphs should have the data points connected? Explain.

Construct Understanding

TRY THIS

Work with a partner.

You will need grid paper.

A. Each of you chooses one of these tasks:

- A sugar cube has a volume of 5 cm³ and a mass of 4 g. Graph the mass of sugar as a function of the number of sugar cubes from 0 to 5 sugar cubes.

- Five cubic centimetres of loose sugar also has a mass of 4 g. Graph the mass of sugar as a function of the volume of sugar from 0 to 25 cm³ of loose sugar.

B. Share your results. How are your graphs alike? How are they different?

C. Work together:

- Identify the dependent variable and independent variable for each function. How did you decide on which axis to graph each variable?

- How did you decide whether to connect the points?

- Are there any restrictions on the domain and range? Explain.

We can represent the function that associates every whole number with its double in several ways.

Using a table of values:

Whole Number, x	Double the Number, y
0	0
1	2
2	4
3	6
4	8

The table continues for all whole numbers.
The domain is the set of whole numbers.
The range is the set of even whole numbers.

Using a graph:
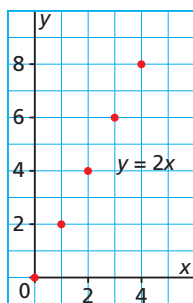
We know the relation $y = 2x$ is a function because each value of x associates with exactly one value of y, and each ordered pair has a different first element.

The *domain* of a function is the set of values of the independent variable; for the graph above, the domain is the x-values.

The *range* of a function is the set of values of the dependent variable; for the graph above, the range is the y-values.

When the domain is restricted to a set of discrete values, the points on the graph are not connected.

A relation that is not a function has two or more ordered pairs with the same first coordinate. So, when the ordered pairs of the relation are plotted on a grid, a vertical line can be drawn to pass through more than one point.	A function has ordered pairs with different first coordinates. So, when the ordered pairs of the function are plotted on a grid, any vertical line drawn will always pass through no more than one point.
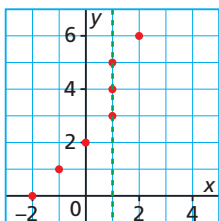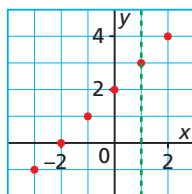
Vertical Line Test for a Function

A graph represents a function when no two points on the graph lie on the same vertical line.
Place a ruler vertically on a graph, then slide the ruler across the graph. If one edge of the ruler always intersects the graph at no more than one point, the graph represents a function.

Which of these graphs represents a function? Justify the answer.

a)   **Height against Shoe Size**
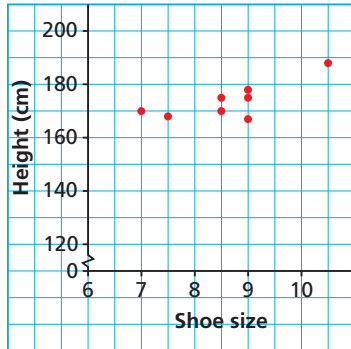
b)   **World Population**
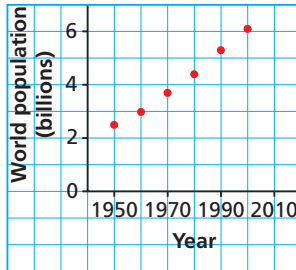
SOLUTION

Use the vertical line test for each graph.

a) This graph does not represent a function because two points lie on the same vertical line.

Height against Shoe Size
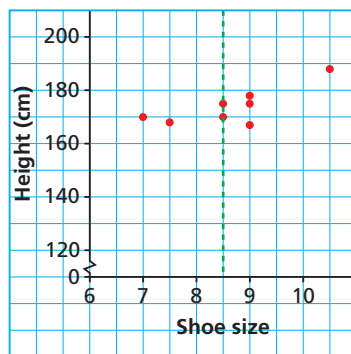
b) This graph does represent a function.
Any vertical line drawn on the graph passes through 0 points or 1 point.

World Population
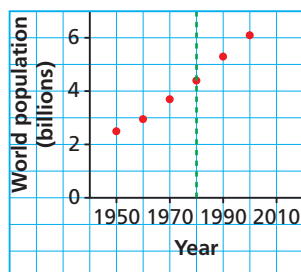
CHECK YOUR UNDERSTANDING

1. Which of these graphs represents a function? Justify your answer.

a)   **Outside Temperature over a 24-h Period**
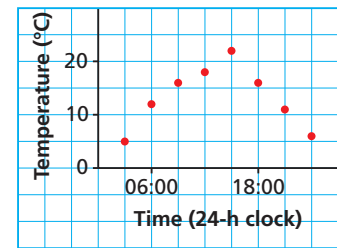
b)   **Masses of Students against Height**
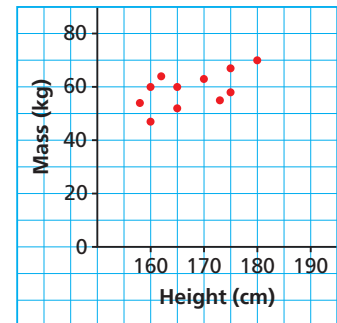
[Answers: a) function
b) not a function]

How does the vertical line test relate to the definition of a function?

Example 2 | **Determining the Domain and Range of the Graph of a Function**

Determine the domain and range of the graph of each function.

a)

b)
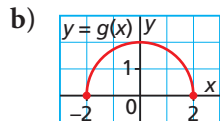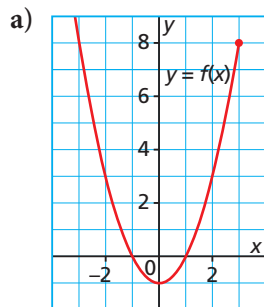
SOLUTION

a) The dot at the right end of the graph indicates that the graph stops at that point.

There is no dot at the left end of the graph, so the graph continues to the left.

The domain is the set of x-values of the function.
Visualize the shadow of the graph on the x-axis.
The domain is the set of all real numbers less than or equal to 3; that is, $x \leq 3$.

The range is the set of y-values of the function.
Visualize the shadow of the graph on the y-axis.
The range is the set of all real numbers greater than or equal to -1; that is, $y \geq -1$.
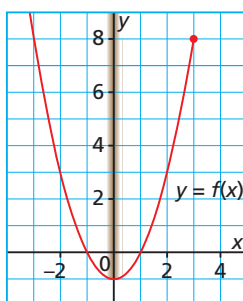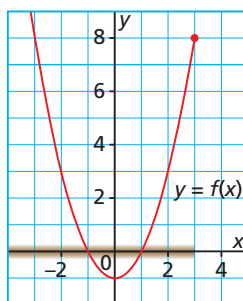
b) The dot at each end of the graph indicates that the graph stops at that point.

The domain is the set of x-values of the function.
Visualize the shadow of the graph on the x-axis.
The domain is the set of real numbers between -2 and 2, including these numbers; that is, $-2 \leq x \leq 2$.

We say: "x is greater than or equal to -2 and less than or equal to 2."
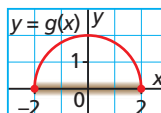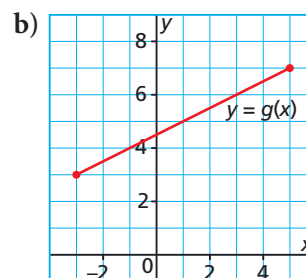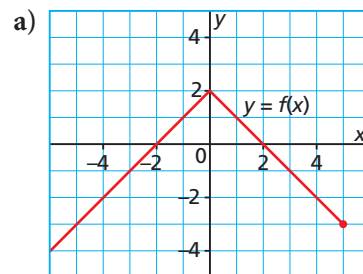
When data are not discrete, we use inequality symbols to indicate the domain and range.

(Solution continues.)

The range is the set of y-values of the function.
Visualize the shadow of the graph on the y-axis.
The range is the set of real numbers between
0 and 2, including these numbers; that is,
$0 \le y \le 2$.
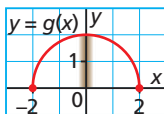
Example 3   Determining the Domain and Range of the Graph of a Situation

This graph shows the number of fishing boats, n, anchored in an inlet in the Queen Charlotte Islands as a function of time, t.

a) Identify the dependent variable and the independent variable. Justify the choices.

b) Why are the points on the graph not connected? Explain.

c) Determine the domain and range of the graph.

Number of Fishing Boats Anchored in an Inlet
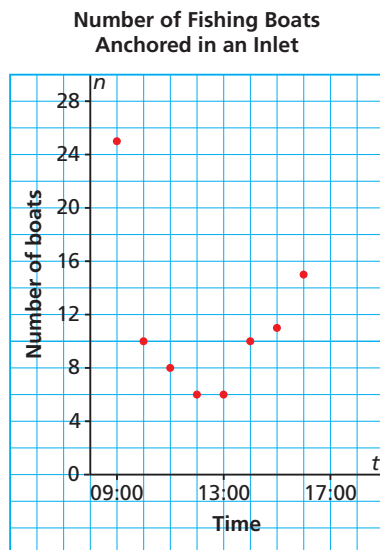
SOLUTION

a) The number of fishing boats is a function of time. Since the number of boats, n, depends on the time of day, the dependent variable is n and the independent variable is t.

b) The points on the graph are not connected because the number of boats is restricted to a whole number. This means that most values between the points are not valid; for example, between 10:00 and 11:00, the number of boats decreases from 10 to 8. We may plot a point at $n = 9$, if we know a corresponding time, but no other point is valid between 10 and 8 because we cannot have a fractional number of boats.

c) The domain is the set of times; that is,
{09:00, 10:00, 11:00, 12:00, 13:00, 14:00, 15:00, 16:00}

The range is the set of the numbers of boats; that is,
{6, 8, 10, 11, 15, 25}

Here is a graph of the function $f(x) = -3x + 7$.
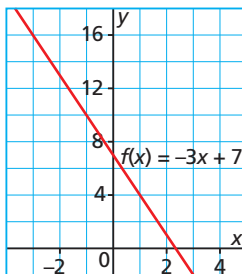
a) Determine the range value when the domain value is -2.

b) Determine the domain value when the range value is 4.

SOLUTION

The domain value is a value of x. The range value is a value of $f(x)$.

a) To determine the value of $f(x)$ when $x = -2$:

Begin at $x = -2$ on the x-axis.

Draw a vertical line to the graph, then a horizontal line to the y-axis.

The line appears to intersect the y-axis at 13.

So, $f(-2) = 13$

When the domain value is -2, the range value is 13.

b) To determine the value of x when $f(x) = 4$:

Since $y = f(x)$, begin at $y = 4$ on the y-axis.

Draw a horizontal line to the graph, then a vertical line to the x-axis.

The line intersects the x-axis at 1.

So, when $f(x) = 4$, $x = 1$

When the range value is 4, the domain value is 1.
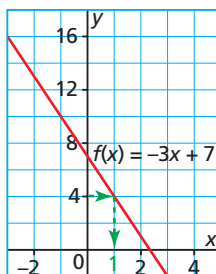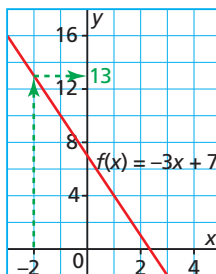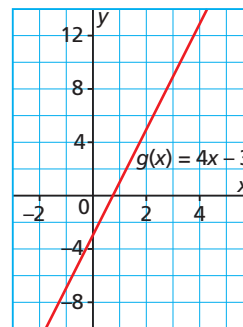
Discuss the Ideas

1. How do you decide whether to connect the points you plot for a graph?

2. What can you tell about the domain and range of a function from its graph?

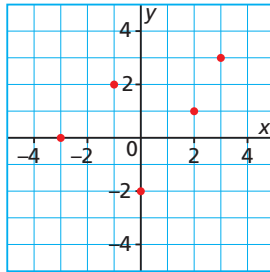3. How can you identify whether a graph represents a function?

Exercises

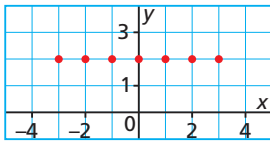**4.** List the domain and the range of the graph of each function.
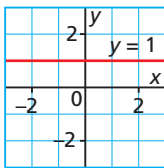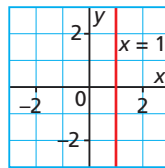
a)
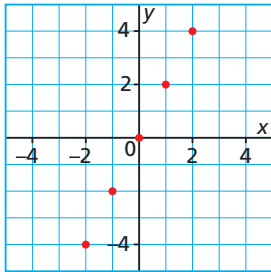
b)

c)

5. How can you tell that each graph in question 4 represents a function?

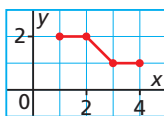**6.** Which of these graphs represents a function? Justify your answer.
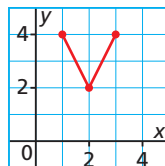
a)

b)

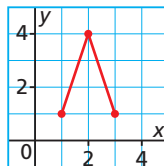**7.** Match the graph of each function to its domain and range listed below.

a)
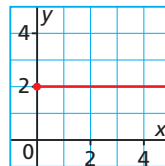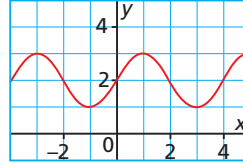

b)

c)

d)

   i) domain: $1 \le x \le 3$; range: $2 \le y \le 4$
   ii) domain: $1 \le x \le 3$; range: $1 \le y \le 4$
   iii) domain: $x \ge 0$; range: $y = 2$
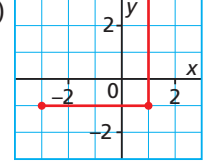   **iv)** domain: $1 \le x \le 4$; range: $1 \le y \le 2$

8. Which of these graphs represents a function? Justify your answer.
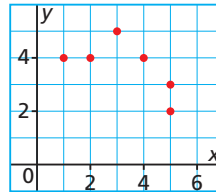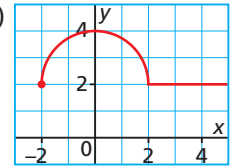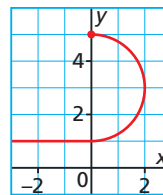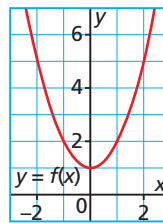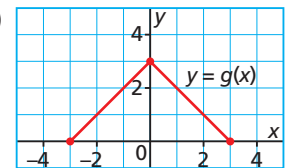Write the domain and range for each graph.

a)

b)

c)

d)

e)

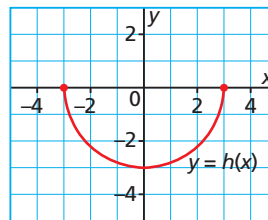**9.** Determine the domain and range of the graph of each function.
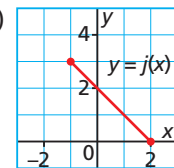
a)

b)

c)

d)

10. Suppose a student drew a graph of each function described below. For which graphs should the student connect the points? Justify your answers.

a) The cost of a custom-made T-shirt is a function of the number of letters on the T-shirt.

b) The altitude of a plane is a function of the time it is in the air.

c) The mass of a baby is a function of her age.

d) The cube root of a real number is a function of the number.

11. a) What do the data in each graph represent?

i)

Graph A

Distance of School Bus from School
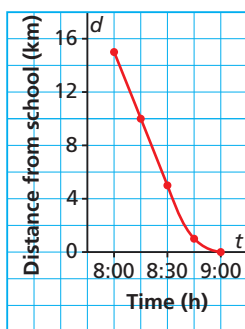
ii)

Graph B

Number of Students on a School Bus
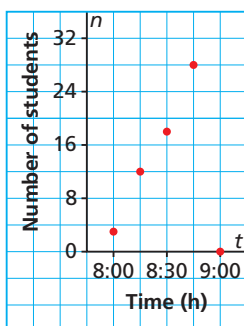
b) Identify the independent and dependent variables.

c) Why are the points connected on one graph but not on the other?

12. When police officers investigate a car crash, they can estimate the speed the car was travelling by measuring the skid distance.

Skid Distance of a Car
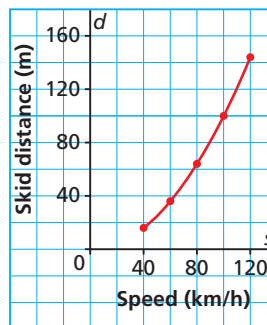
a) Why are the points on the graph connected?

b) Estimate the domain and range of the graph. Are there any restrictions on the domain and range? Explain.

13. This graph shows the number of cars, n, in the school parking lot as a function of time, t.

Number of Cars in the School Parking Lot
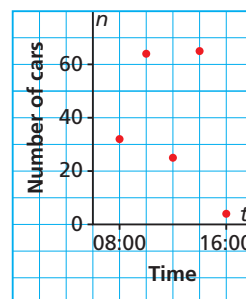
a) Identify the independent and dependent variables. Justify your choices.

b) Why are the points on the graph not connected?

c) Estimate the domain and range of the graph. Are there any restrictions on the domain and range? Explain.

14. Paulatuuq is north of the Arctic Circle. The table shows the number of hours, h, the sun is above the horizon every 60 days from January 1st, which is day 0.

Day	h
0	0
60	9.7
120	18.5
180	24.0
240	15.9
300	7.4
360	0

   a) Identify the independent variable and the dependent variable. Justify your choices.
   b) Graph the data in the table. Did you connect the points? Why or why not?
   c) Use the table of values and the graph to explain why this relation is a function.

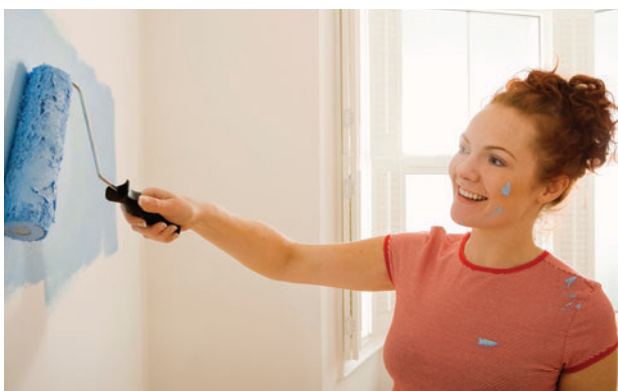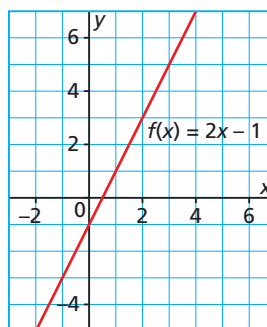**15.** One litre of latex paint covers approximately 8.5 m² and costs $12.
   a) Copy and complete this table.

Volume of Paint, p (L)	0	2	4	6	8
Cost, c ($)		0	24		
Area Covered, A (m²)		0	17		

   b) Graph the area covered as a function of the volume of paint.
   c) Graph the area covered as a function of the cost.
   d) Write the domain and range of the functions in parts b and c.

16. This is a graph of the function $f(x) = 2x - 1$.

   a) Determine the range value when the domain value is 0.
   b) Determine the domain value when the range value is 5.

17. This is a graph of the function $g(x) = -x + 3$.
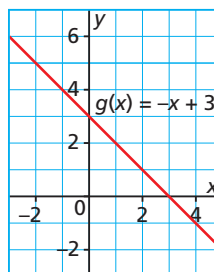
   a) Determine the range value when the domain value is -2.
   b) Determine the domain value when the range value is 0.

18. Draw a graph of a function on a grid. Write the domain and range of the function. Exchange graphs with a classmate, and check that the domain and range of your classmate's graph are correct. If they are not, correct them, then explain your corrections to your classmate.

19. Sketch a graph of a function that has each domain and range.
   a) domain: $-2 \le x \le 3$; range: $1 \le y \le 5$
   b) domain: $x \ge 1$; range: $-1 \le y \le 1$

20. One planetary year is the time it takes for a planet to travel once around the sun. Since the planets take different times to travel around the sun, one year on each planet is different. The distance from Earth to the sun is 1 astronomical unit. Other distances in the solar system are compared to the distance from Earth to the sun.

	Earth	Jupiter	Saturn	Uranus
Distance from Sun (astronomical units)	1	5	10	19
Planetary Year (Earth years)	1	12	29	84

a) Graph planetary year as a function of distance from the sun. Did you connect the points? Explain.
b) Write the domain and range of this function.

C

21. This table shows the costs to send letters within Canada in 2009.

Mass of Letter	Cost ($)
Up to 30 g	0.54
Greater than 30 g and up to 50 g	0.98
Greater than 50 g and up to 100 g	1.18
Greater than 100 g and up to 200 g	1.96
Greater than 200 g and up to 500 g	2.75

a) Graph the cost of sending a letter as a function of its mass. Did you connect the points? Explain.
b) Write the domain and range of this function.

22. A hospital patient has his temperature taken every hour.

Patient's Temperature against Time
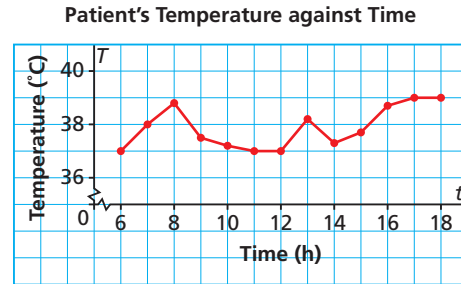
Should the points have been connected? Give reasons for your answer.

23. Is this statement true?
A measure of time can be any real number, so any graph with time as its independent variable should have its points connected.
Explain your answer with examples.

24. Payment scheme 1: A person receives 1¢ on day 1, then each day the payment is doubled.
Payment Scheme 2: A person receives $10 each day.
For both payments, the total money received is a function of the number of days.
a) Make a table of values for each payment scheme.
b) Graph the data.
c) Which payment scheme would you choose if you were receiving the money for 30 days? Explain.

Reflect

Generalize and explain rules for determining whether a graph represents a function. How do you determine the domain and range of a function from its graph? Include examples in your explanation.

Connections

Concept map centered on "Graphs of relations and functions" with the following connected nodes:

- Draw a graph for a given situation
  - How are the axes labelled?
  - How are the key points plotted?
  - Where does the graph start and end?
- Interpret a given graph
  - What do the axes represent?
  - What do the key points represent?
  - Where does the graph start and end?
  - Does the graph increase, decrease, or stay constant?
  - Are the points joined?
- Use technology
- Use grid paper
- Is the graph a function?
  - Use the vertical line test
  - Identify the independent and dependent variables
    - From a situation
    - From a graph
      - Independent variable labels the horizontal axis
      - Dependent variable labels the vertical axis
  - Identify the domain and range
    - Can you extend the domain and range?
    - Use projections (shadows) onto each axis

Concept Development

- **In Lesson 5.3**
  - You applied what you know about functions to interpret graphs that represent different situations.
  - You applied what you know about functions to sketch graphs that represent different situations.

- **In Lesson 5.4**
  - You generated data for a relation, then graphed and analyzed the data.

- **In Lesson 5.5**
  - You used the vertical line test on graphs to identify functions.
  - You identified the independent and dependent variables of a function.
  - You graphed tables of values for functions and identified their domains and ranges.
  - You connected points on a graph if all real-number values of the variables were permitted.

Assess Your Understanding

1. Copy the graph below. Choose labels for each axis, then describe a situation the graph could represent. Justify your description.
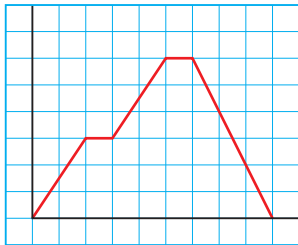

5.4

2. a) Use technology or grid paper to graph these data for people up to the age of 18.

Age (years)	Mass (kg)
14	45
14	50
15	56
15	64
17	65
18	90

   b) Should you join the points? Explain your reasoning.

   c) What are the domain and range of these data?

   d) Suppose data for more people, up to the age of 18, with different masses were graphed. Would there be any restrictions on the domain and range? If your answer is yes, state the restrictions. If your answer is no, explain why no restrictions exist.

5.5

3. Which graphs represent functions? Justify your answer. Write the domain and range of each graph.

a)

b)

c)
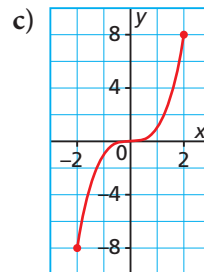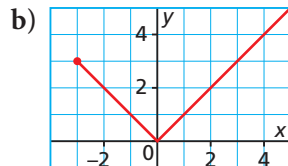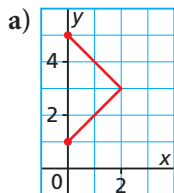
5.6 Properties of Linear Relations

Make Connections

The table of values and graph show the cost of a pizza with up to 5 extra toppings.

Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75
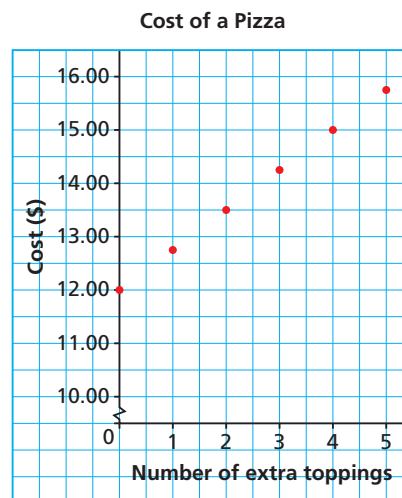
Cost of a Pizza

What patterns do you see in the table?

Write a rule for the pattern that relates the cost of a pizza to the number of its toppings.

How are the patterns in the table shown in the graph?

How can you tell from the table that the graph represents a linear relation?
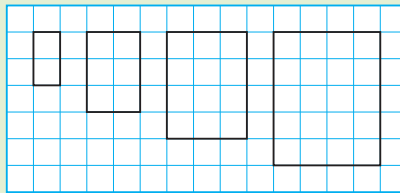
Construct Understanding

TRY THIS

Work with a partner.

You will need 1-cm grid paper.

Use this pattern of rectangles.
This pattern continues.

A. Draw the next two rectangles in the pattern.
Copy and complete each table of values for the 6 rectangles.

Width of Rectangle (cm)	Area (cm^2)
1	
2	

Width of Rectangle (cm)	Perimeter (cm)
1	
2	

B. Which table of values represents a linear relation? How can you tell?

C. Graph the data in each table of values.
Does each graph represent a linear relation?
How do you know?

The cost for a car rental is $60, plus $20 for every 100 km driven.
The independent variable is the distance driven and the dependent variable
is the cost.
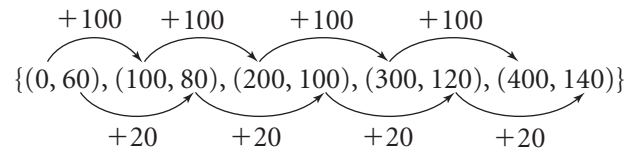We can identify that this is a linear relation in different ways.

- a table of values

Independent variable ⟶

Distance (km)	Cost ($)
0	60
100	80
200	100
300	120
400	140

⟵ Dependent variable

+100 → +20
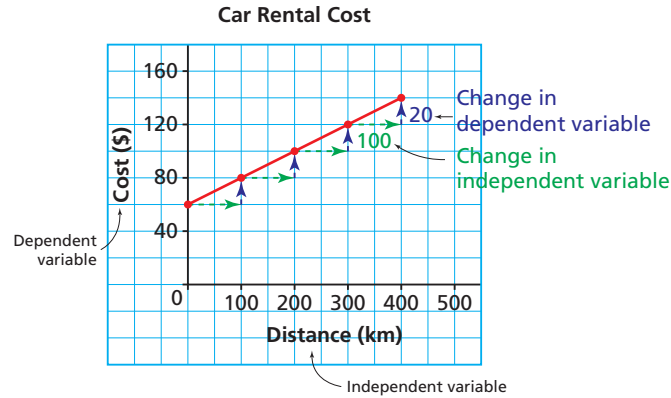+100 → +20
+100 → +20
+100 → +20

For a linear relation, a constant change in the independent variable results in
a constant change in the dependent variable.

Why is it important that
the ordered pairs are
listed so their first
elements are in
numerical order?

- a set of ordered pairs

$$\overbrace{}^{+100}\ \overbrace{}^{+100}\ \overbrace{}^{+100}\ \overbrace{}^{+100}$$

$$\{(0, 60), (100, 80), (200, 100), (300, 120), (400, 140)\}$$

$$\underbrace{}_{+20}\ \underbrace{}_{+20}\ \underbrace{}_{+20}\ \underbrace{}_{+20}$$

- a graph

The graph of a linear
relation is a straight line.

Car Rental Cost

We can use each representation above to calculate the **rate of change**.
The rate of change can be expressed as a fraction:

$$\frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{\$20}{100\ \text{km}}$$

$$= \$0.20/\text{km}$$

The rate of change is $0.20/km; that is, for each additional 1 km driven, the
rental cost increases by 20¢. The rate of change is constant for a linear relation.

We can determine the rate of change from the equation that represents
the linear function.
Let the cost be C dollars and the distance driven be d kilometres.
An equation for this linear function is:
$C = 0.20d + 60$

       — initial amount
       — independent variable
       — rate of change
       — dependent variable

Example 1

Example 1 | **Determining whether a Table of Values Represents a Linear Relation**

Which table of values represents a linear relation? Justify the answer.

a) The relation between temperature in degrees Celsius, C, and temperature in degrees Fahrenheit, F

C	F
0	32
5	41
10	50
15	59
20	68

b) The relation between the current, I amps, and power, P watts, in an electrical circuit

I	P
0	0
5	75
10	300
15	675
20	1200

SOLUTION

The terms in the first column are in numerical order. So, calculate the change in each variable.

a)

C	Change in C	F	Change in F
0		32	
5	5 − 0 = 5	41	41 − 32 = 9
10	10 − 5 = 5	50	50 − 41 = 9
15	15 − 10 = 5	59	59 − 50 = 9
20	20 − 15 = 5	68	68 − 59 = 9

Since the changes in both variables are constant, the table of values represents a linear relation.

b)

I	Change in I	P	Change in P
0		0	
5	5 − 0 = 5	75	75 − 0 = 75
10	10 − 5 = 5	300	300 − 75 = 225
15	15 − 10 = 5	675	675 − 300 = 375
20	20 − 15 = 5	1200	1200 − 675 = 525

The changes in I are constant, but the changes in P are not constant. So, the table of values does not represent a linear relation.

CHECK YOUR UNDERSTANDING

1. Which table of values represents a linear relation? Justify your answer.

a) The relation between the number of bacteria in a culture, n, and time, t minutes.

t	n
0	1
20	2
40	4
60	8
80	16
100	32

b) The relation between the amount of goods and services tax charged, T dollars, and the amount of the purchase, A dollars

A	T
60	3
120	6
180	9
240	12
300	15

[Answers: a) not linear b) linear]

What other strategies could you use to check whether each table of values represents a linear relation?

When an equation is written using the variables x and y, x represents the independent variable and y represents the dependent variable.

Example 2 — Determining whether an Equation Represents a Linear Relation

a) Graph each equation.

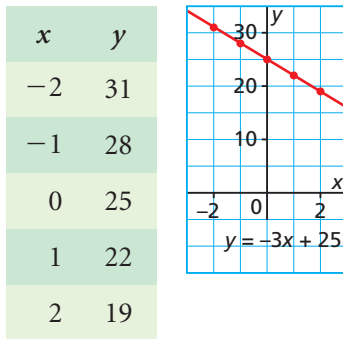   **i)** $y = -3x + 25$ **ii)** $y = 2x^2 + 5$

    iii) $y = 5$ **iv)** $x = 1$

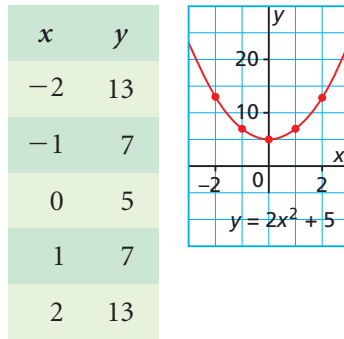**b)** Which equations in part a represent linear relations? How do you know?
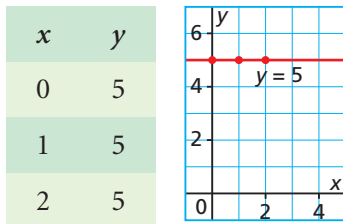
SOLUTION

a) Create a table of values, then graph the relation.

    i) $y = -3x + 25$

x	y
-2	31
-1	28
0	25
1	22
2	19

    ii) $y = 2x^2 + 5$

x	y
-2	13
-1	7
0	5
1	7
2	13

    iii) $y = 5$

x	y
0	5
1	5
2	5

    iv) $x = 1$

x	y
1	-1
1	0
1	1
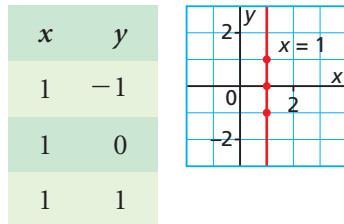
b) The graphs in parts i, iii, and iv are straight lines, so their equations represent linear relations; that is, $y = -3x + 25$, $y = 5$, and $x = 1$.
The graph in part ii is not a straight line, so its equation does not represent a linear relation.

CHECK YOUR UNDERSTANDING

2. a) Graph each equation.
    i) $x = -2$
    ii) $y = x + 25$
    iii) $y = 25$
    iv) $y = x^2 + 25$

b) Which equations in part a represent linear relations? How do you know?

[Answers: b) $x = -2$; $y = x + 25$; $y = 25$]

Example 3   Identifying a Linear Relation

Which relation is linear? Justify the answer.

a) A new car is purchased for $24 000. Every year, the value of the car decreases by 15%. The value is related to time.

b) For a service call, an electrician charges a $75 flat rate, plus $50 for each hour he works. The total cost for service is related to time.

SOLUTION

Create a table of values, then check to see if the relation is linear.

a) Every year, the value decreases by 15%.
The value of the car is:
100% − 15% = 85% of its previous value
So, multiply each value by 0.85.

Time (years)	Value ($)
0	24 000
1	20 400
2	17 340
3	14 739

+1 — −3600
+1 — −3060
+1 — −2601

There is a constant change of 1 in the 1st column, but the differences in the 2nd column are not constant. So, the relation is not linear.

b) After the first hour, the cost increases by $50 per hour.

Time (h)	Cost ($)
0	75
1	125
2	175
3	225
4	275

+1 — +50
+1 — +50
+1 — +50
+1 — +50

There is a constant change of 1 in the 1st column and a constant change of 50 in the 2nd column, so the relation is linear.

A water tank on a farm near Swift Current, Saskatchewan, holds 6000 L.
Graph A represents the tank being filled at a constant rate.
Graph B represents the tank being emptied at a constant rate.

Graph A
Filling a Water Tank
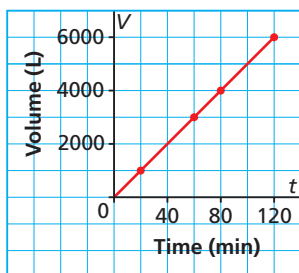
Graph B
Emptying a Water Tank
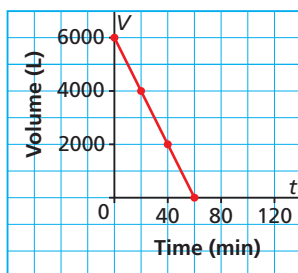
a) Identify the independent and dependent variables.

b) Determine the rate of change of each relation, then describe what it represents.

SOLUTION

For Graph A

a) The independent variable is the time, t.
The dependent variable is the volume, V.

b) Choose two points on the line. Calculate the change in each variable from one point to the other.

Graph A
Filling a Water Tank

Volume increases by 1000 L

Time increases by 20 min

Change in volume:
$4000 \text{ L} - 3000 \text{ L} = 1000 \text{ L}$

Change in time:
$80 \text{ min} - 60 \text{ min} = 20 \text{ min}$

Rate of change: $\dfrac{1000 \text{ L}}{20 \text{ min}} = 50 \text{ L/min}$

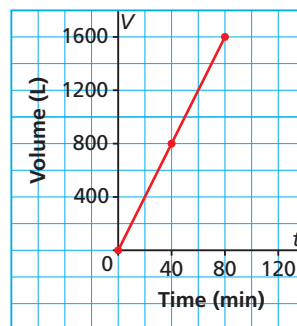The rate of change is positive so the volume is increasing with time.
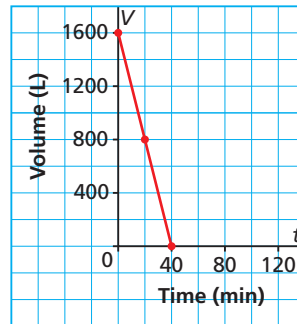Every minute, 50 L of water are added to the tank.
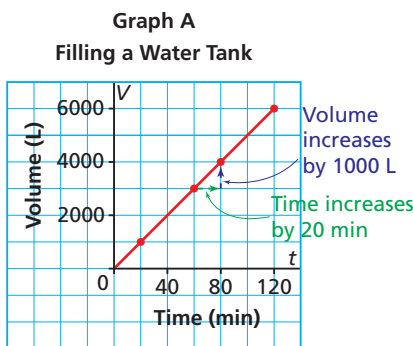
CHECK YOUR UNDERSTANDING

4. A hot tub contains 1600 L of water. Graph A represents the hot tub being filled at a constant rate. Graph B represents the hot tub being emptied at a constant rate.

Graph A
Filling a Hot Tub


Graph B
Emptying a Hot Tub

a) Identify the dependent and independent variables.

b) Determine the rate of change of each relation, then describe what it represents.

[Answers: Graph A a) V, t b) 20 L/min
Graph B a) V, t b) -40 L/min]

For Graph B

a) The independent variable is the time, t.
The dependent variable is the volume, V.

b) Choose two points on the line.
Calculate the change in each variable from one point to the other.

Graph B
Emptying a Water Tank

Volume decreases by 2000 L
Time increases by 20 min
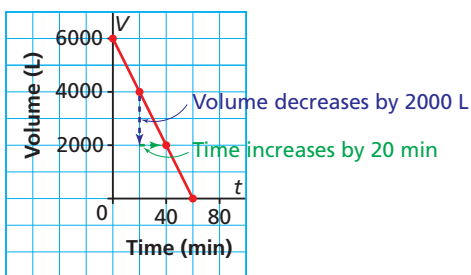
Change in volume: $2000 \text{ L} - 4000 \text{ L} = -2000 \text{ L}$

Change in time: $40 \text{ min} - 20 \text{ min} = 20 \text{ min}$

Rate of change: $\dfrac{-2000 \text{ L}}{20 \text{ min}} = -100 \text{ L/min}$

The rate of change is negative so the volume is decreasing with time.
Every minute, 100 L of water are removed from the tank.

Discuss the Ideas

1. How can you tell from each format whether a relation is linear?
- a description in words
- a set of ordered pairs
- a table of values
- an equation
- a graph

2. What is "rate of change"? How can you use each format in question 1 to determine the rate of change of a linear relation?

Exercises

3. Which tables of values represent linear relations? Explain your answers.

a)

Time (min)	Distance (m)
0	10
2	50
4	90
6	130

b)

Time (s)	Speed (m/s)
0	10
1	20
2	40
3	80

c)

Speed (m/s)	Time (s)
15	7.5
10	5
5	2.5
0	0

d)

Distance (m)	Speed (m/s)
4	2
16	4
1	1
9	3

4. Which sets of ordered pairs represent linear relations? Explain your answers.
a) $\{(3, 11), (5, 9), (7, 7), (9, 5)\}$
b) $\{(-2, 3), (0, 1), (2, -3), (4, -7)\}$
c) $\{(1, 1), (1, 3), (2, 1), (2, 3)\}$

5. Which graphs represent linear relations? How do you know?

a)

b)

c)

d)
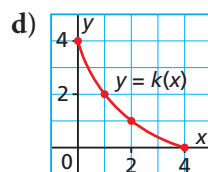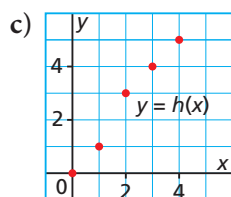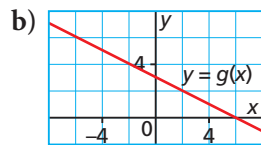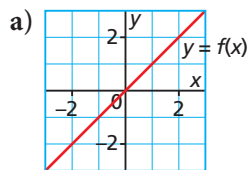
6. a) Create a table of values when necessary, then graph each relation.
    i) $y = 2x + 8$ **ii)** $y = 0.5x + 12$
    iii) $y = x^2 + 8$ **iv)** $y = 2x$
    v) $x = 7$ **vi)** $x + y = 6$
  b) Which equations in part a represent linear relations? How do you know?

7. For each relation below:
    i) Identify the dependent and independent variables.
    ii) Use the table of values to determine whether the relation is linear.
    iii) If the relation is linear, determine its rate of change.
  a) The distance required for a car to come to a complete stop after its brakes are applied is the *braking distance*. The braking distance, *d* metres, is related to the speed of the car, *s* kilometres per hour, when the brakes are first applied.

s (km/h)	d (m)
50	13
60	20
70	27
80	35

  b) The altitude of a plane, *a* metres, is related to the time, *t* minutes, that has elapsed since it started its descent.

t (min)	a (m)
0	12 000
2	11 600
4	11 200
6	10 800
8	10 400

8. In a hot-air balloon, a chart shows how the distance to the horizon, d kilometres, is related to the height of the balloon, h metres.

h (m)	d (km)
5	8
10	11
30	20
50	25
100	36

a) Graph these data.
b) Is the relation linear? What strategy did you use?

9. Earth rotates through approximately 360° every 24 h. The set of ordered pairs below describes the rotation. The first coordinate is the time in hours, and the second coordinate is the approximate angle of rotation in degrees. Describe two strategies you could use to determine if this relation is linear.
$\{(0, 0), (6, 90), (12, 180), (18, 270), (24, 360)\}$

10. Sophie and 4 of her friends plan a trip to the Edmonton Chante for one night. The hotel room is $95 for the first 2 people, plus $10 for each additional person in the room. The total cost is related to the number of people. Is the relation linear? How do you know?

11. A skydiver jumps from an altitude of 3600 m. For the first 12 s, her height in metres above the ground is described by this set of ordered pairs:
$\{(0, 3600), (4, 3526), (8, 3353.5), (12, 3147.5)\}$
For the next 21 s, her height above the ground is described by this set of ordered pairs:
$\{(15, 2988.5), (21, 2670.5), (27, 2352.5), (33, 2034.5)\}$
Determine whether either set of ordered pairs represents a linear relation. Explain.

12. The cost, C dollars, to rent a hall for a banquet is given by the equation $C = 550 + 15n$, where n represents the number of people attending the banquet.
a) Explain why the equation represents a linear relation.
b) State the rate of change. What does it represent?

13. A safety flare is shot upward from the top of a cliff 200 m above sea level. An equation for the height of the flare, d metres, above sea level t seconds after the flare is fired, is given by the equation $d = -4.9t^2 + 153.2t + 200$. Describe two strategies you could use to determine whether this relation is linear.

14. This graph represents Jerome's long distance phone call to his pen pal in Nunavut. Jerome is charged a constant rate.

The Cost of Jerome's Phone Call
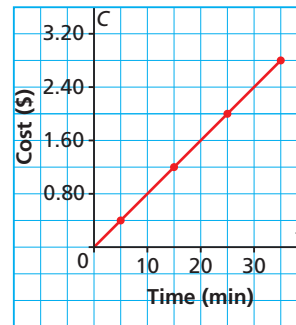
a) Identify the dependent and independent variables.
b) Determine the rate of change, then describe what it represents.

15. Kashala takes a cross-country trip from her home in Lethbridge through the United States. In Illinois, she drives on a toll highway. This graph represents the cost of Kashala's drive on the toll highway. She is charged a constant amount at each toll booth and she starts with US$10 in change. Determine the rate of change, then describe what it represents.

Kashala's Drive on the Toll Highway
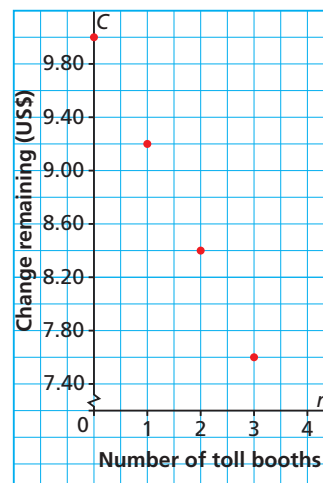
16. Match each description of a linear relation with its equation and set of ordered pairs below. Explain your choices.

a) The amount a person earns is related to her hourly wage.

b) The cost of a banquet is related to a flat fee plus an amount for each person who attends.

c) The volume of gas in a car's gas tank is related to the distance driven since the time when the tank was filled.

Equation 1: $y = 500 + 40x$

Equation 2: $y = 35 - 0.06x$

Equation 3: $y = 20x$

Set A: $\{(100, 29), (200, 23), (300, 17), (400, 11)\}$

Set B: $\{(1, 20), (5, 100), (10, 200), (15, 300)\}$

Set C: $\{(0, 500), (40, 2100), (80, 3700), (100, 4500)\}$

17. a) Which situations represent linear relations? Explain how you know.

i) A hang glider starts her descent at an altitude of 2000 m. She descends at a constant speed to an altitude of 1500 m in 10 min.

ii) A population of bacteria triples every hour for 4 h.

iii) A taxi service charges a $5 flat fee plus $2 for each kilometre travelled.

iv) The cost to print each yearbook is $5. There is a start up fee of $500 to set up the printing press.

v) An investment increases in value by 12% each year.

b) For each linear relation in part a, identify:
- the dependent and independent variables
- the rate of change and explain what it represents

18. Identify the measurement formulas that represent linear relations. Explain how you know.

a) Perimeter, P, of an equilateral triangle with side length s: $P = 3s$

b) Surface area, A, of a cube with edge length s: $A = 6s^2$

c) Volume, V, of a sphere with radius r:
$$V = \frac{4}{3}\pi r^3$$

d) Circumference, C, of a circle with diameter d: $C = \pi d$

e) Area, A, of a circle with radius r: $A = \pi r^2$

19. Here are two equations that can be used to model the value, V dollars, of a $24\ 000 truck as it depreciates over n years:
$V = 24\ 000 - 2000n$ and $V = 24\ 000(0.2^n)$

a) Which equation represents a linear relation? Justify your answer.

b) For the linear relation, state the rate of change. What does it represent?

20. You can estimate the distance in kilometres between you and a distant storm by measuring the time in seconds between seeing a lightning flash and hearing the thunder, then dividing by 3. This works because sound travels at approximately 0.3 km/s. Is this relation between distance and time linear? Justify your answer.

21. A berry patch is to be harvested. Is the relation between the time it will take to harvest the patch and the number of pickers needed linear? Justify your answer.

22. Which statements are true? Use examples to justify your answers.

a) A relation described by exactly two ordered pairs is always linear.

b) An equation of the form $Ax + By = C$ for non-zero constants, A, B, and C, always represents a linear function.

c) An equation of the form $y = Cx^2$ for a non-zero constant C, always represents a linear function.

d) An equation of the form $x = C$ for a constant C, always represents a linear relation.

e) A linear relation is always a linear function.

Reflect

List three different strategies you can use to tell whether a relation is linear. Include an example with each strategy.

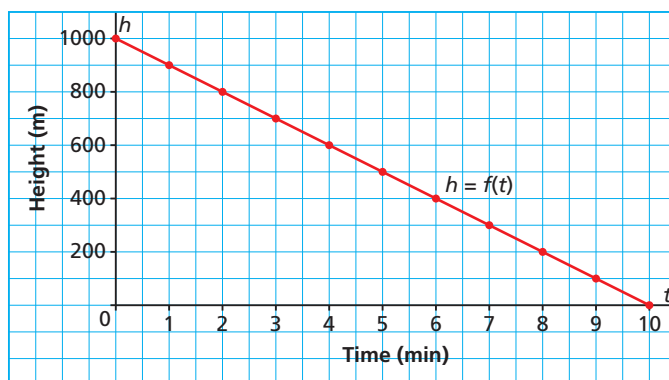# 5.7 Interpreting Graphs of Linear Functions

Make Connections

Float planes fly into remote lakes in Canada's Northern wilderness areas for ecotourism. This graph shows the height of a float plane above a lake as the plane descends to land.

Height of a Float Plane

Where does the graph intersect the vertical axis? What does this point represent?

Where does the graph intersect the horizontal axis? What does this point represent?

What is the rate of change for this graph? What does it represent?

Construct Understanding

Work in a group.

You will need grid paper.

Dogsled tours are run between Armstrong cabin and Irving cabin. The cabins are 100 km apart.

Dogsled team 1 travels at an average speed of 20 km/h and starts its tour at Armstrong cabin.

Dogsled team 2 travels at an average speed of 25 km/h and starts its tour at Irving cabin.

One pair of students chooses team 1 and the other pair chooses team 2.

A. Copy and complete the table to show the distance from Irving cabin at different times on the tour.

Team 1

Time (h)	Distance from Irving Cabin (km)
0	100
1	

Team 2

Time (h)	Distance from Irving Cabin (km)
0	0
1	

B. Draw a graph to show the distance from Irving cabin as a function of time.

C. Share your results with the other pair of students.

- How are the graphs the same? How are they different?
- Identify where each graph intersects the vertical and horizontal axes. What do these points represent?
- Determine the rate of change for each graph. What does it represent?
- What are the domain and range for each graph?

Any graph of a line that is not vertical represents a function. We call these functions **linear functions**.

Each graph below shows the temperature, T degrees Celsius, as a function of time, t hours, for two locations.

Temperature in Location A
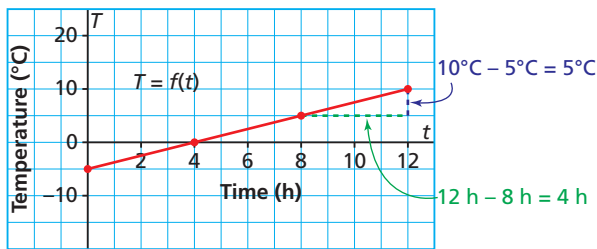
The point where the graph intersects the horizontal axis has coordinates $(4, 0)$. The **horizontal intercept** is 4. This point of intersection represents the time, after 4 h, when the temperature is 0°C.

The point where the graph intersects the vertical axis has coordinates $(0, -5)$. The **vertical intercept** is -5. This point of intersection represents the initial temperature, -5°C.

The *domain* is: $0 \le t \le 12$
The *range* is: $-5 \le T \le 10$

The *rate of change* is: $\dfrac{\text{change in } T}{\text{change in } t} = \dfrac{5°C}{4\,h}$

$$= 1.25°C/h$$

The rate of change is positive because the temperature is increasing over time.

Temperature in Location B
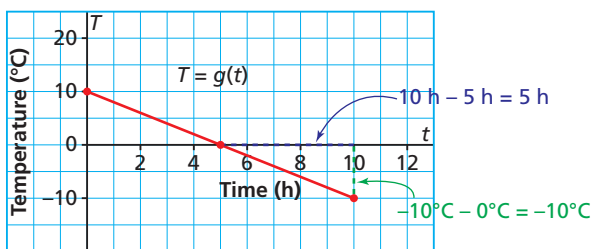
The point where the graph intersects the horizontal axis has coordinates $(5, 0)$. The *horizontal intercept* is 5. This point of intersection represents the time, after 5 h, when the temperature is 0°C.

The point where the graph intersects the vertical axis has coordinates $(0, 10)$. The *vertical intercept* is 10. This point of intersection represents the initial temperature, 10°C.

The *domain* is: $0 \le t \le 10$
The *range* is: $-10 \le T \le 10$

The *rate of change* is: $\dfrac{\text{change in } T}{\text{change in } t} = \dfrac{-10°C}{5\,h}$

$$= -2°C/h$$

The rate of change is negative because the temperature is decreasing over time.

This graph shows the fuel consumption of a scooter with a full tank of gas at the beginning of a journey.

Volume of Gas in a Scooter

a) Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.

b) What are the domain and range of this function?

SOLUTION

a) On the vertical axis, the point of intersection has coordinates $(0, 8)$. The vertical intercept is 8. This point of intersection represents the volume of gas in the tank when the distance travelled is 0 km; that is, the capacity of the gas tank: 8 L

On the horizontal axis, the point of intersection has coordinates $(200, 0)$. The horizontal intercept is 200. This point of intersection is the distance travelled until the volume of gas is 0 L; that is, the distance the scooter can travel on a full tank of gas: 200 km

b) The domain is the set of possible values of the distance travelled:
$0 \leq d \leq 200$
The range is the set of possible values of the volume of fuel:
$0 \leq V \leq 8$

CHECK YOUR UNDERSTANDING

1. This graph shows how the height of a burning candle changes with time.

Height of a Burning Candle
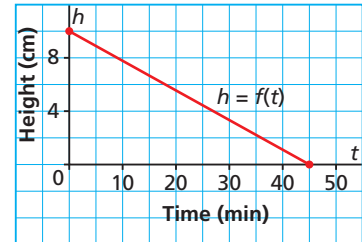
a) Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.

b) What are the domain and range of this function?

[Answers: a) $(0, 10)$, 10; $(45, 0)$, 45 b) domain: $0 \leq t \leq 45$; range: $0 \leq h \leq 10$]

Are there any restrictions on the domain and range? Explain.

What is the fuel consumption in litres per 100 km?

We can use the intercepts to graph a linear function written in function notation.

To determine the y-intercept, evaluate $f(x)$ when $x = 0$; that is, evaluate $f(0)$.

To determine the x-intercept, determine the value of x when $f(x) = 0$.
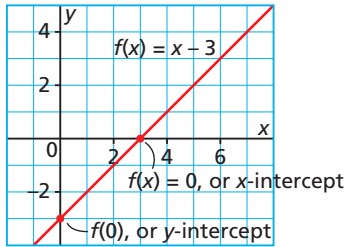
Example 2 Sketching a Graph of a Linear Function in Function Notation

Sketch a graph of the linear function $f(x) = -2x + 7$.

SOLUTION

$f(x) = -2x + 7$

Since the function is linear, its graph is a straight line.

Determine the y-intercept:

When $x = 0$,

$f(0) = -2(0) + 7$

$f(0) = 7$

Determine the x-intercept:

When $f(x) = 0$,

$0 = -2x + 7$

$-7 = -2x + 7 - 7$

$-7 = -2x$

$x = \dfrac{-7}{-2}$

$x = \dfrac{7}{2}$

Determine the coordinates of a third point on the graph.

When $x = 1$,

$f(1) = -2(1) + 7$

$f(1) = 5$

Plot the points $(0, 7)$, $\left(\dfrac{7}{2}, 0\right)$,

and $(1, 5)$, then draw a line

through them.
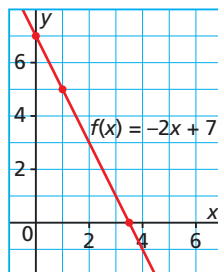
CHECK YOUR UNDERSTANDING

2. Sketch a graph of the linear function $f(x) = 4x - 3$.

Answer:
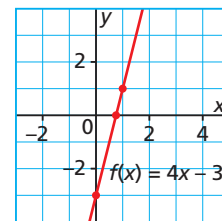
What other strategy could you use to graph the function? Which strategy would be more efficient?

Which graph has a rate of change of $\frac{1}{2}$ and a vertical intercept of 6? Justify the answer.

a)

b)

3. Which graph has a rate of change of -5 and a vertical intercept of 100? Justify your answer.

a)
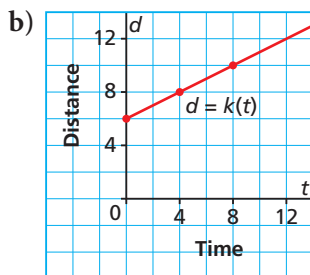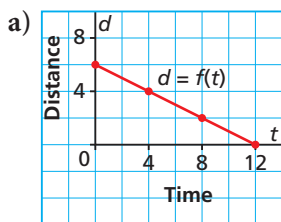
SOLUTION

a) The graph of $d = f(t)$ has a vertical intercept of 6.
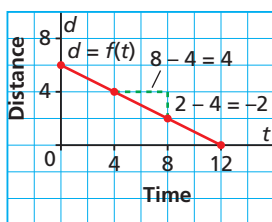
The rate of change is: $\dfrac{-2}{4} = -\dfrac{1}{2}$

So, it is not the correct graph.
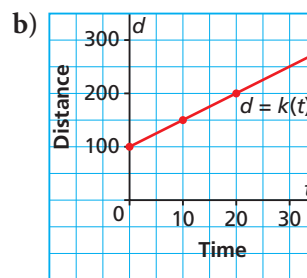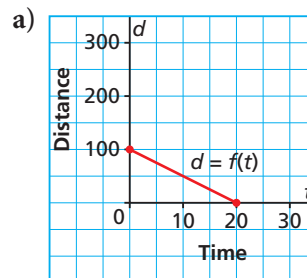
b) The graph of $d = k(t)$ has a vertical intercept of 6.
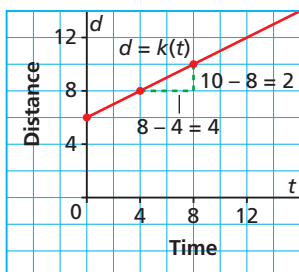
The rate of change is: $\dfrac{2}{4} = \dfrac{1}{2}$

So, this is the correct graph.

b)

[Answer: the graph in part a]

Example 4   **Solving a Problem Involving a Linear Function**

This graph shows the cost of publishing a school yearbook for Collège Louis-Riel in Winnipeg.

Cost of Publishing a Yearbook
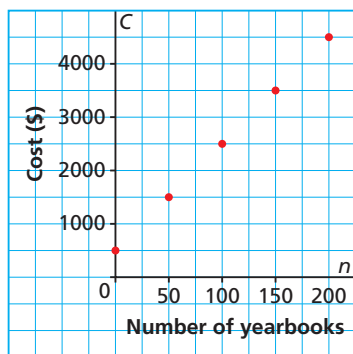
The budget for publishing costs is $4200. What is the maximum number of books that can be printed?

SOLUTIONS

Method 1

To estimate the number of yearbooks that can be printed for $4200, use the graph.

From 4200 on the C-axis, draw a horizontal line to the graph, then a vertical line to the n-axis.

Cost of Publishing a Yearbook
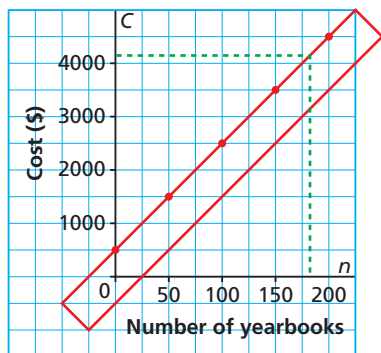
Use a straightedge to help.

From the graph, about 180 yearbooks can be printed.

(Solution continues.)

4. This graph shows the total cost for a house call by an electrician for up to 6 h work.

Cost of an Electrician's House Call
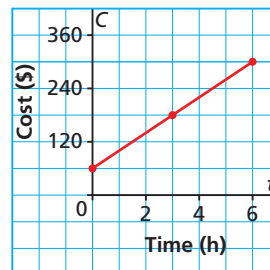
The electrician charges $190 to complete a job. For how many hours did she work?

[Answer: $3\frac{1}{4}$ h]

Why are the points on this graph not joined?

What are the domain and range of this function?

Method 2

The set-up cost is the cost when the number of books printed is 0.
This is the vertical intercept of the graph, which is 500.
The set-up cost is $500.

The increase in cost for each additional book printed is the rate of
change of the function. Determine the change in each variable.

Cost of Publishing a Yearbook

$2500 − $1500 = $1000

100 books − 50 books = 50 books
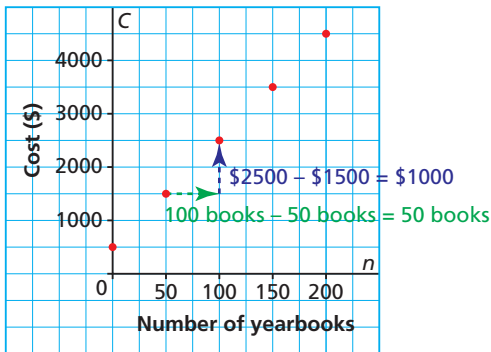
The graph shows that for every 50 books printed, the cost
increases by $1000.

The rate of change is: $\dfrac{\$1000}{50\,\text{books}} = \$20/\text{book}$

The increase in cost for each additional book published is $20.

An equation that represents this situation is: $C = 20n + 500$

To determine the maximum number of yearbooks that can be
printed, use the equation:

$$
\begin{aligned}
C &= 20n + 500 \qquad \text{Substitute: } C = 4200 \\
4200 &= 20n + 500 \qquad \text{Solve for } n. \\
4200 - 500 &= 20n + 500 - 500 \\
3700 &= 20n \\
\frac{3700}{20} &= \frac{20n}{20} \\
185 &= n
\end{aligned}
$$

The maximum number of yearbooks that can be printed is 185.

What is an advantage of using
each method?

1. What information do the vertical and horizontal intercepts provide about a linear function? Use an example to explain.

2. How can you tell from a graph whether a linear function has a positive or negative rate of change?

3. When a situation can be described by a linear function, why doesn't it matter which pair of points you choose to determine the rate of change?

Exercises

A

4. Each graph below shows distance, d kilometres, as a function of time, t hours. For each graph:
   i) Determine the vertical and horizontal intercepts. Write the coordinates of the points where the graph intersects the axes.
   ii) Determine the rate of change.
   iii) Determine the domain and range.

a)

b)
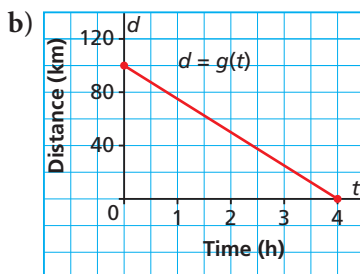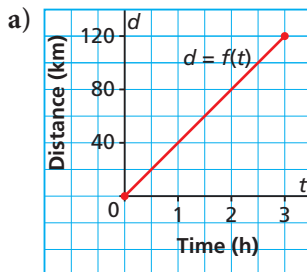
5. Each graph shows the altitude, A feet, of a small plane as a function of time, t minutes. For each graph:
   i) Determine the vertical intercept. Write the coordinates of the point where the graph intersects the axis.
   ii) Determine the rate of change.
   iii) Determine the domain and range.

a)

b)
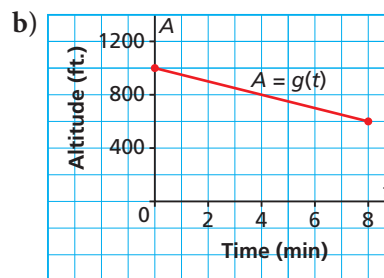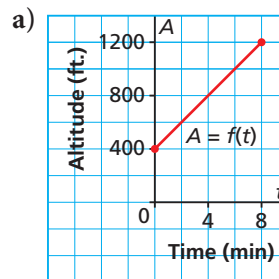
B

6. Sketch a graph of each linear function.
   a) $f(x) = 4x + 3$
   b) $g(x) = -3x + 5$
   c) $h(x) = 9x - 2$
   d) $k(x) = -5x - 2$

7. This graph shows the area, A square metres, that paint covers as a function of its volume, V litres.
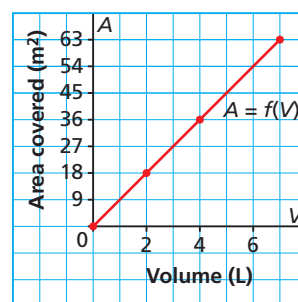
   a) What is the rate of change? What does it represent?
   b) What area is covered by 6 L of paint?
   c) What volume of paint would cover 45 m²?

8. The graphs below show the temperature, T degrees Celsius, as a function of time, t hours, at different locations.
   a) Which graph has a rate of change of 5°C/h and a vertical intercept of $-10°C$?
   b) Which graph has a rate of change of $-10°C/h$ and a vertical intercept of 20°C?

   i)

   ii)

   iii)

   iv)
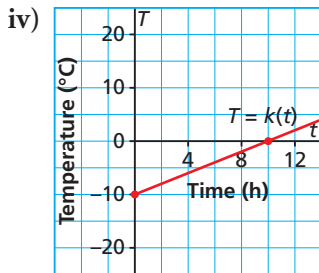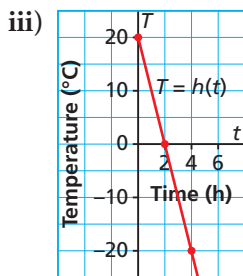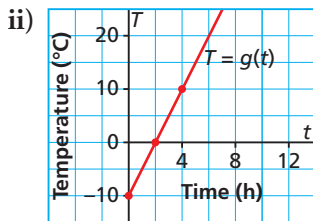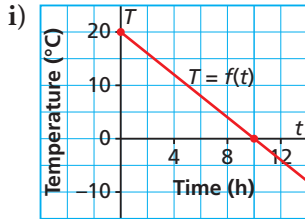
9. St. Adolphe, Manitoba, is located in the flood plain of the Red River. To help prevent flooding, backhoes were used to build dikes around houses and farms in the town. This graph shows the labour costs for running a backhoe.

Cost of Running a Backhoe
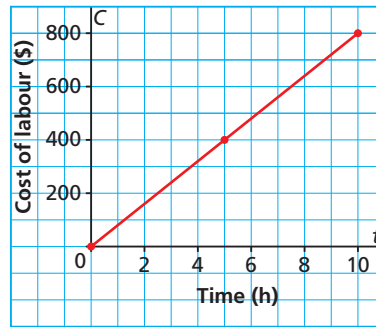
   a) Determine the vertical and horizontal intercepts. Write the coordinates of the point where the graph intersects the axes. Describe what the point represents.
   b) Determine the rate of change. What does it represent?
   c) Write the domain and range.
   d) What is the cost to run the backhoe for 7 h?
   e) For how many hours is the backhoe run when the cost is $360?

10. This graph shows the cost for a cab at Eagle Taxi Cabs. The cost, C dollars, is a function of the distance travelled, d kilometres.
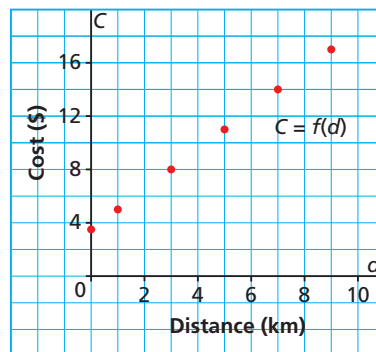
   a) Determine the rate of change. What does it represent?
   b) What is the cost when the distance is 7 km?
   c) What is the distance when the cost is $9.50?

11. A Smart car and an SUV have full fuel tanks, and both cars are driven on city roads until their tanks are nearly empty. The graphs show the fuel consumption for each vehicle.

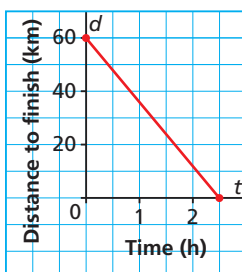**Fuel Consumption of a Smart Car**
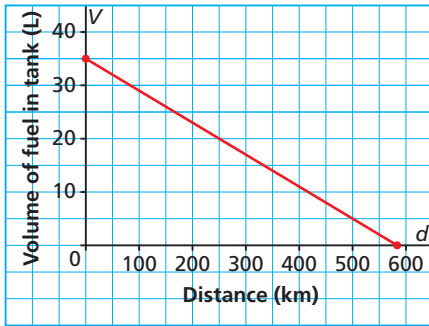
Fuel Consumption of an SUV
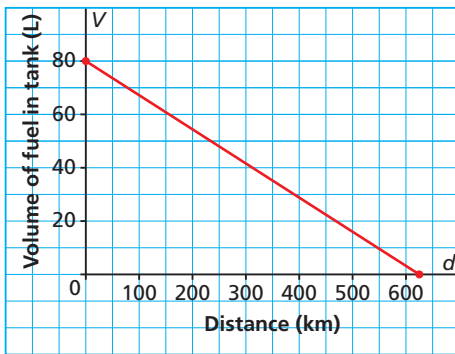
Use the graphs to explain why the Smart car is more economical to drive than the SUV.

12. This graph shows the distance to the finish line, d kilometres, as a function of time, t hours, for one dogsled in a race near Churchill, Manitoba.

Dogsled Race

a) What was the length of time it took the dogsled to finish the race?
b) What was the average speed of the dogsled?
c) How long was the race in kilometres?
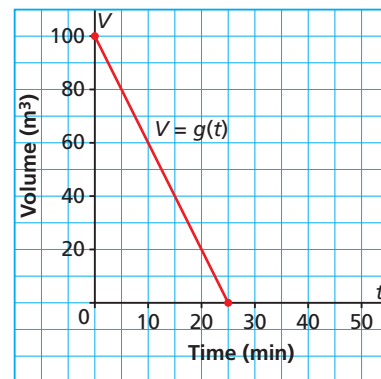d) What time did it take for the dogsled to complete $\dfrac{2}{3}$ of the race?

13. The capacity of each of 2 fuel storage tanks is 100 m³. Graph A represents the volume of fuel in one tank as a function of time as the tank is filled. Graph B represents the volume of fuel in another tank as a function of time as the tank is emptied.
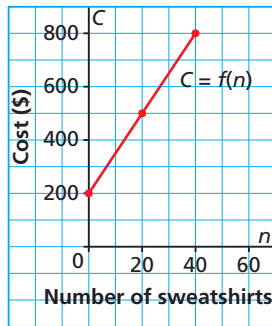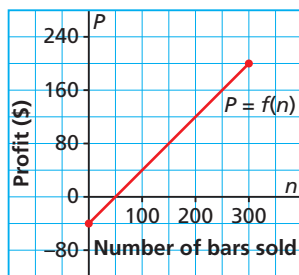
Graph A
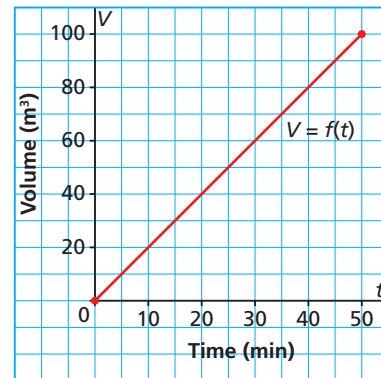
Graph B

a) Does it take longer to fill the empty tank or empty the full tank? How do you know?
b) In the time it takes for one tank to be half empty, about how much fuel would be in a tank that was being filled from empty?

14. Ballenas School places an order for school sweatshirts with its logo of a killer whale on the back. This graph shows the cost of the sweatshirts, C dollars, as a function of the number ordered, n.

a) The number of sweatshirts cannot be a fraction or decimal. Why do you think the points on the graph are joined?

b) i) About how many sweatshirts can be bought for $700?

ii) Suppose one more sweatshirt was ordered. What would be the increase in cost?

15. Sketch a graph of each linear function for positive values of the independent variable.

a) $f(x) = 5 - 2.5x$ b) $g(t) = 85t$

c) $h(n) = 750 + 55n$ d) $V(d) = 55 - 0.08d$

16. Northlands School Outdoor Club had a fundraiser to help purchase snowshoes. The club had 300 power bars to sell. This graph shows the profit made from selling power bars.

a) What is the profit on each bar sold? How do you know?

b) Determine the intercepts. What does each represent?

c) Describe the domain and range for the function. Why would you not want to list all the values in the range?

17. This graph shows the recommended maximum heart rate of a person, R beats per minute, as a function of her or his age, a years, for a stress test.

a) Why are there no intercepts on this graph?

b) What is the rate of change? What does it represent?

c) At what age is the recommended maximum heart rate 120 beats/min?

d) What is the approximate recommended maximum heart rate for a person aged 70?

C

18. Two graphs that relate two real numbers x and y in different ways are shown below. For each graph:

i) State the x- and y-intercepts.

ii) Use the intercepts to describe how x and y are related.

a)

b)
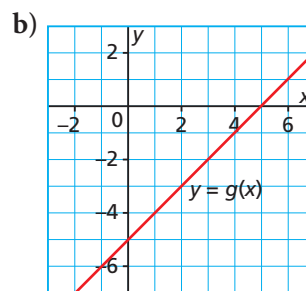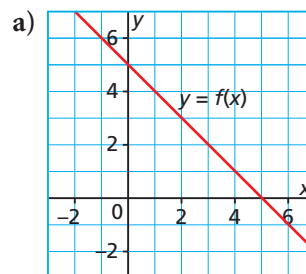
19. a) Sketch a graph of the linear function
$d = f(t)$ that satisfies these conditions:
$f(1.5) = 127.5$ and $f(3.5) = 297.5$
b) Determine $f(5)$.
c) Determine t when $f(t) = 212.5$.
d) Suggest a context for this linear function.

20. The distance between Parksville and the Duke Point Ferry Terminal on Vancouver Island is 50 km. A person drives from Parksville to the ferry terminal.

Distance from Parksville and to Duke Point

Distance to Duke Point (km) / Distance from Parksville (km)
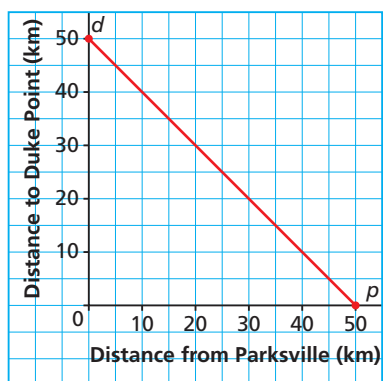
a) What do the intercepts represent? Why are they equal?
b) What is the rate of change? Why does it not have units? What does it indicate?
c) How would interchanging the dependent and independent variables change the graph?
d) Suppose the distance between two towns A and B is k kilometres. Describe the graph of the function, "Distance to A as a function of distance from B". State the intercepts, domain and range, and the rate of change.

Reflect

Explain why knowing the intercepts and the rate of change of the graph of a linear function may be helpful when you solve problems. Include examples in your explanation.

 THE WORLD OF MATH

Historical Moment: Theano
Theano was one of the first known woman mathematicians. Her husband was Pythagoras, perhaps the most famous mathematician of all time. Theano lived in the 6th century B.C.E. in what is now southern Italy. She wrote many articles on mathematics, as well as on physics, medicine, astronomy, and child psychology. Her most famous work was on the development of the golden ratio and the golden rectangle.

STUDY GUIDE

CONCEPT SUMMARY

Big Ideas	Applying the Big Ideas
■ A relation associates the elements of one set with the elements of another set.	This means that: ■ A relation may be represented as: a rule, a table, a set of ordered pairs, an arrow diagram, and a graph. The set of first elements is the domain and the set of related second elements is the range.
■ A function is a special type of relation for which each element of the first set is associated with a unique element of the second set.	■ For a function, each element of the domain is associated with exactly one element of the range.
■ A linear function has a constant rate of change and its graph is a non-vertical straight line.	■ For a linear function, a constant change in the independent variable results in a constant change in the dependent variable, and any vertical line drawn through the graph intersects the graph at no more than one point.

Reflect on the Chapter

- What is a relation? What is a function? Create a graphic organizer to show their common characteristics, and those that are unique.
- How can the properties of linear functions be used to solve real-world problems? Include examples with your explanation.

SKILLS SUMMARY

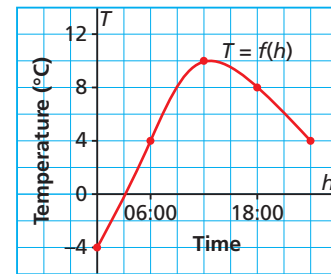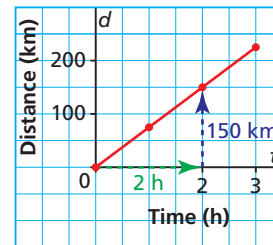| Skill	Description	Example
Determine the domain and range of a function. [5.2, 5.4, 5.5, 5.7]	The domain is the set of first elements of the ordered pairs. The range is the set of second elements. For a graph, the domain is the set of values of the independent variable. The range is the set of values of the dependent variable.	$\{(-1, 3), (0, 5), (1, 7), (2, 9), (3, 11)\}$ For this set of ordered pairs, the domain is: $\{-1, 0, 1, 2, 3\}$; the range is: $\{3, 5, 7, 9, 11\}$ For the graph below: The domain is all possible times in one day. The range is: $-4 \le T \le 10$
Determine the rate of change of the graph of a linear function. [5.6, 5.7]	The rate of change is: $$\frac{\text{change in dependent variable}}{\text{change in independent variable}}$$ The rate of change is positive when the graph goes up to the right. The rate of change is negative when the graph goes down to the right.	**Distance against Time** The rate of change is: $$\frac{150 \text{ km}}{2 \text{ h}} = 75 \text{ km/h}$$
Determine the intercepts of the graph of a linear function. [5.7]	The x-intercept is the value of x when y or $f(x)$ is 0. The y-intercept is the value of y when x is 0.	For the linear function $f(x) = -2x + 5$, When $f(x) = 0$: $0 = -2x + 5$ $2x = 5$ $x = 2.5$ The x-intercept is 2.5. When $x = 0$: $f(0) = -2(0) + 5$ $f(0) = 5$ The y-intercept is 5.

1. This table shows some Northwest Coast artists and their cultural heritage.

Artist	Heritage
Bob Dempsey	Tlingit
Dorothy Grant	Haida
Bill Helin	Tsimshian
John Joseph	Squamish
Judith P. Morgan	Gitxsan
Bill Reid	Haida
Susan Point	Salish

a) Describe the relation in words.
b) Represent this relation:
    i) as a set of ordered pairs
    ii) as an arrow diagram

2. Here is a list of some chemical elements and their atomic numbers:
hydrogen (1), oxygen (8), iron (26), chlorine (17), carbon (6), silver (47)
For each association below, use these data to represent a relation in different ways.
a) has an atomic number of
b) is the atomic number of

5.2

3. Which sets of ordered pairs represent functions? What strategies did you use to find out?
a) $\{(4, 3), (4, 2), (4, 1), (4, 0)\}$
b) $\{(2, 4), (-2, 4), (3, 9), (-3, 9)\}$
c) $\{(2, 8), (3, 12), (4, 16), (5, 20)\}$
d) $\{(5, 5), (5, -5), (-5, 5), (-5, -5)\}$

4. Write in function notation.
a) $y = -4x + 9$ b) $C = 12n + 75$
c) $D = -20t + 150$ d) $P = 4s$

5. The function $P(n) = 5n - 300$ describes the profit, P dollars, for a school dance when n students attend.

a) Write the function as an equation in 2 variables.
b) Identify the independent variable and the dependent variable. Justify your choices.
c) Determine the value of $P(150)$. What does this number represent?
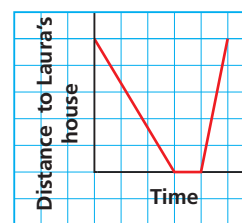d) Determine the value of n when $P(n) = 700$. What does this number represent?

5.3

6. a) Laura cycles home from school, then walks back to school. Which graph best matches this situation? Explain your choice.
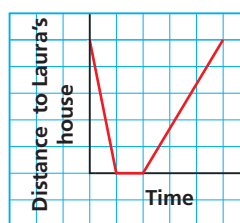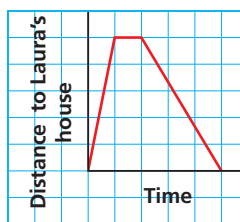

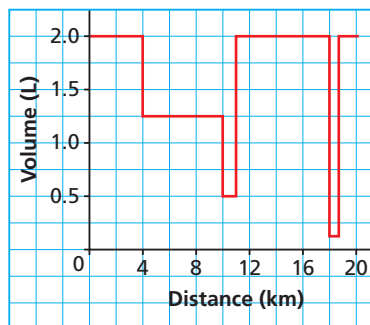**Graph A**   **Graph B**

Graph C   **Graph D**

b) Choose one of the graphs in part a that did not describe Laura's journey. Describe a possible situation for the graph.

7. This graph shows the volume of water in Liam's flask as he hikes the Trans Canada trail.

Water in Liam's Flask

a) Describe what is happening for each line segment of the graph.
b) How many times did Liam fill his flask?

c) How much water was in Liam's flask at the start of his hike?

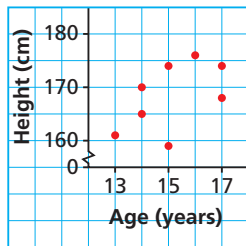**d)** Identify the dependent and independent variables.

8. The data below show how the temperature of boiling water as it cools is related to time.

a) Graph the data. Did you join the points? Why or why not?

b) Does the graph represent a function? How can you tell?

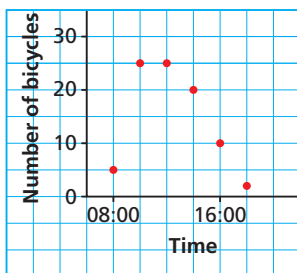| Time (min)	Temperature (°C)
0	89
5	78
10	69
15	62
20	57
25	53
30	50

9. Which of these graphs represents a function? Justify your answer.

Write the domain and range for each graph.

a) Heights and Ages of 8 Students

b) Number of Bicycles at School

10. For the graphs below:

a) What does each graph represent?

b) Identify the independent and dependent variables.

c) Write the domain and range for each graph. Estimate when necessary. Are there any restrictions on the domain and range? Explain.

d) Why are the points joined on one graph but not on the other?

i) Graph A

ii) Graph B
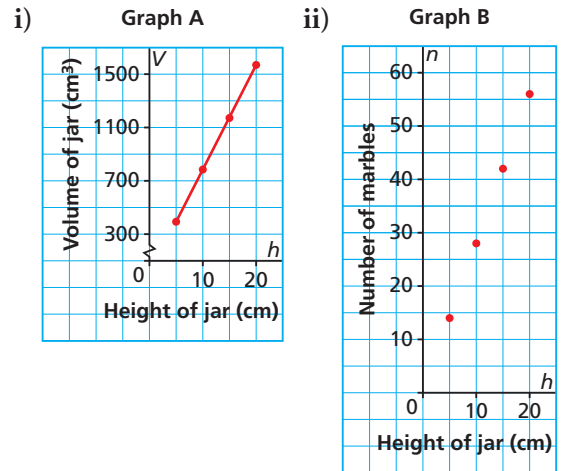
11. This is a graph of the function $f(x) = -3x + 1$.
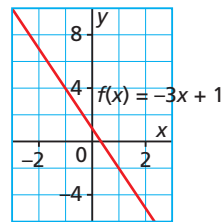
a) Determine the range value when the domain value is 1.

b) Determine the domain value when the range value is 4.

12. Sketch a graph of a function that has each domain and range.

a) domain: $-1 \leq x \leq 5$; range: $0 \leq y \leq 3$

b) domain: $x \leq 1$; range: $-2 \leq y \leq 2$

13. Which sets of ordered pairs represent linear relations? Explain your answers.

a) $\{(1, 5), (5, 5), (9, 5), (13, 5)\}$

b) $\{(1, 2), (1, 4), (1, 6), (1, 8)\}$

c) $\{(-2, -3), (-1, -2), (2, 1), (4, -3)\}$

14. a) For each equation, create a table of values when necessary, then graph the relation.

  i) $x = 3$
  ii) $y = 2x^2 + 3$
  iii) $y = 2x + 3$
  iv) $y = 3$
  v) $y = 3x$
  vi) $x + y = 3$

 **b)** Which equations in part a represent linear relations? How do you know?

15. Isabelle manages her diabetes by taking insulin to control her blood sugar. The number of units of insulin taken, N, is given by the equation $N = \dfrac{1}{15} g$, where g represents the number of grams of carbohydrates consumed.

  a) Explain why the equation represents a linear relation.
 **b)** State the rate of change. What does it represent?

16. This graph shows the distance, d metres, travelled by Jadan on her bicycle as a function of the number of wheel revolutions, n, as she rode from Whitehorse to the Grey Mountain Road lookout in the Yukon.

Jadan's Bicycle Trip
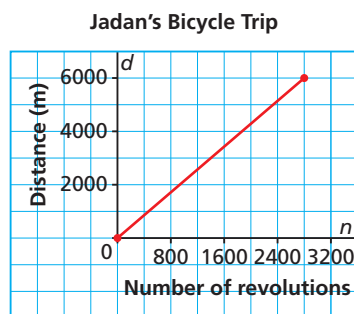
  a) How far was Jadan from the lookout when she started her bicycle trip?
  b) Write the domain and range.
  c) Determine the rate of change. What does it represent?
  d) Use your answer to part c to determine the diameter of a bicycle wheel.

17. These graphs show the temperature, T degrees Celsius, as a function of time, t hours. Match each graph with its vertical intercept and rate of change.
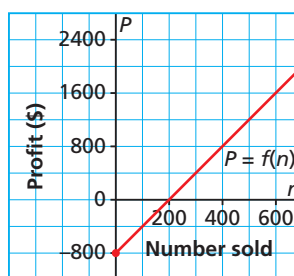
a)

b)

c)
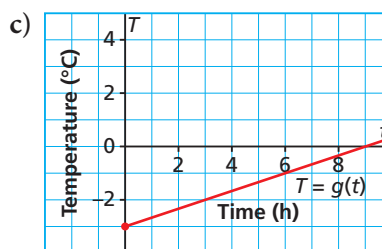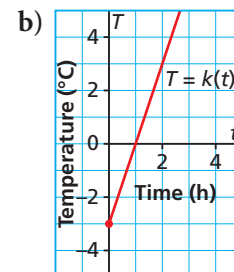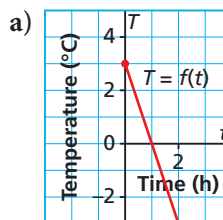
  i) $-3°C; \dfrac{1}{3}°C/h$

  ii) $3°C; -3°C/h$

  iii) $-3°C; 3°C/h$

18. This graph shows the profit, P dollars, on a company's sale of n baseball caps.

  a) How many baseball caps have to be sold before the company begins to make a profit?
  b) What is the profit on the sale of each baseball cap?
  c) How many caps have to be sold to make each profit?
    i) $600   **ii)** $1200
 **d)** In part c, when the profit doubles why does the number of baseballs caps sold not double?

For questions 1 and 2, choose the correct answer: A, B, C, or D

1. For the function $f(x) = 3 - 6x$, what is the value of $f(-3)$?

A. 1 **B.** 21 **C.** -15 **D.** 0

2. Which equation does *not* represent a linear function?

A. $f(x) = 5$ **B.** $f(x) = 5x$ **C.** $f(x) = 5x^2$ **D.** $f(x) = -5$

3. For each relation represented below:
　　　i) State whether it is a function and how you know.
　　ii) If the relation is a function:
　　　　State its domain and range.
　　　　Represent the function in a different way.
　　　　State whether it is a linear function and how you know.
　　iii) If the relation is a linear function:
　　　　Identify the dependent and independent variables.
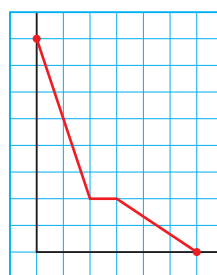　　　　Determine the rate of change.

a) $\{(2, 5), (-3, 6), (1, 5), (-1, 4), (0, 2)\}$

b)

n	s
2	4
-1	1
1	1
-3	9

c)
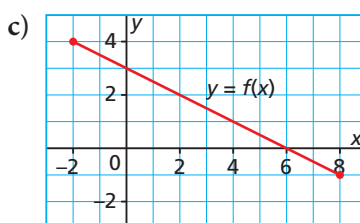
4. Describe a possible situation for this graph. Label the axes and give the graph a title. Justify your description.

5. This table of values shows how the time to cook a turkey is related to its mass.
　a) Why is this relation a function?
　b) Identify the dependent and the independent variables. Justify your choice.
　c) Graph the data. Did you connect the points? Explain.
　d) Determine the domain and range of the graph. Could you extend the graph? Identify and explain any restrictions on the domain and range. Explain.
　e) Determine the rate of change for this function. What does it represent?
　f) For how long should you cook a turkey with mass 7 kg?

Mass (kg)	Time (h)
4	2.5
6	3.0
8	3.5
10	4.0